Heterogeneous Economies: Implications for Inequality and Financial Stability.

by

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Contents

To my parents, Anna and Stratos.

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Declarations

This work has not been submitted for a degree at another university.

Chapters 1 and 2 are my own work. Chapter 3 is collaborative work with Alex Karlis, Spyros Terovitis and Matthew Turner. The basic idea was mine and was further developed through joint discussions, the simulations and proofs were done by A. Karlis, while the write-up and final editing was shared equally among the four of us.

Abstract

In the first chapter we explore the relationship between income inequality and the Utilitarian ethic in a dynamic environment with endogenous preferences. Classical Utilitarians, like Bentham, believed that utilitarian principles are compatible with egalitarian ones. Although this claim is not uncontroversial, this relation holds for a utilitarian distribution of a given good among people, with identical concave utilities and exogenously set preferences. This idea breaks down if the preferences are different. In this paper we allow for endogenous preferences influenced by the existence of habits. We show how the inclusion of habit formation, studied in a dynamic environment, has egalitarian implications for a classical utilitarian distribution. Based on this result we are able to argue that Bentham's positive views of decreasing inequality due to different consumption habits are consistent with his normative views regarding distribution.

The second chapter explores the question of whether long-term income inequality consistent with equality of opportunity (EOp) ethic. In order to provide an answer we study the effectiveness of intergenerational EOp policies in an environment with two social groups and infinite generations of individuals, where the outcomes of one generation define the circumstances of the next. Circumstances in this paper have to do either with different preferences among individuals from different social groups or with both resources and preferences due to these resources. We show that in the former case EOp policies reduce inequality and also the EOp policy is the same as the Utilitarian one. In the latter case, inequality is not reduced and its level depends on the relative population of the two social groups.

The third chapter studies an economy where privately informed hedge funds trade a risky asset in order to exploit potential mispricings. Hedge funds are allowed to have access to credit, by using their risky assets as collateral. We analyse the role of the degree of heterogeneity among hedge funds's demand for the risky asset in the emergence of clustering of defaults. We find that fire-sales caused by margin calls is a necessary, yet not a sufficient condition for defaults to be clustered. We show that when the degree of heterogeneity is sufficiently high, poorly performing hedge funds are able to obtain a higher than usual market share at the end of the leverage cycle, which leads to an improvement of their performance. Consequently, their survival time is prolonged, increasing the probability of them remaining in operation until the downturn of the next leverage cycle. This leads to the increase of the probability of poorly and high-performing hedge funds to default in sync at a later time, and thus the probability of collective defaults.

Chapter 1

Utilitarianism and Habits of Opulence and Poverty

1.1 Introduction

> "In a nation which prospers by agriculture, manufactures, and commerce, there is a continual progress towards equality. This will be the result of different habits formed by opulence and poverty" [Bentham, 2001b, p.313]

According to classical utilitarianism, the end of government should be "the maximisation of the happiness of the whole community under consideration" [Bentham, 1993, p.5]. The utilitarian approach thus defined incorporates some powerful intuitions: it has the virtue of simplicity and it provides a clear cut answer to a number of issues in the realms of personal morality as well as in social and political philosophy. This probably explains why it is one of the most prominent and widely adopted approaches in normative economics.

Yet both its philosophical foundations and some of its key implications have been criticised[1]. In the context of social choice theory one fundamental feature of utilitarianism has attracted critical attention, namely its potentially very undesirable distributive implications. On the one hand, the utilitarian planner is definitionally indifferent between alternative allocations given a *certain level of aggregate utility*. Call this the *inequality indifference critique*. On the other hand, the very maximisation of total utility may require an extremely unequal allocation of both resources

[1] For detailed criticisms of utilitarianism as a theory of the good, see for example Smart and Williams [1973] and the contributions in Sen and Williams [1982].

and utility, especially if agents have different preferences. Call this the *inequality generation critique*.

Utilitarians have rejected these criticisms, or at least significantly deflated their relevance and have traditionally argued that their view was compatible with what they have historically considered to be one of the fundamental characteristics of a well ordered society, namely equality. The utilitarian response to the inequality indifference critique is at the formal and theoretical level. It focuses on the possibility of refinements of the utilitarian view requiring an egalitarian distribution of a *given amount of aggregate utility.* Bentham [2001b] for example, proposed that equality along with security, subsistence and abundance should be one of the objectives of a government. Sidgwick argued that Utilitarianism necessarily relied on a principle of equality in order to determine the best mode of distributing utility:

> "In all such cases, therefore, it becomes practically important to ask whether any mode of distributing a given quantum of happiness is better than any other. Now the Utilitarian formula seems to supply no answer to this question: at least we have to supplement the principle of seeking the greatest happiness of the whole by some principle of Just or Right distribution of this happiness. The principle which most Utilitarians have either tacitly or expressly adopted is that of pure equality-as given in Bentham's formula, "everybody to count for one, and nobody for more than one." And this principle seems the only one which does not need a special justification; for, as we saw, it must be reasonable to treat any one man in the same way as any other, if there be no reason apparent for treating him differently." [Sidgwick, 1907, p. 417]

The utilitarian response to the inequality generation critique involves an argument that can be interpreted either as an empirical or as a normative claim. Given that agents hold similar preferences, and given decreasing marginal utility (say, of income) the optimal utilitarian distribution will be roughly egalitarian. The empirical interpretation is especially prominent in classical hedonistic utilitarian approaches, in which only the agents' actual subjective preferences (whatever they are) matter. As Samuelson put it "If people are all alike, or potentially all alike in the longer runs as environmentalists like Bentham and Mill believed, total U is maximised by an equal distribution of income (achieved by ideal lump-sum redistributions that have no distorting substitution effect)" [Samuelson, 1964, p. 417] In Bentham, for example, the egalitarian implications of utilitarianism derive from what he calls the *axioms of moral and political pathology*, namely those empirical generalisations that are “expressive of the connection between such occurrences as are continually taking place or are liable to take place, and the pleasures and pains which are respectively the results of them” [Parekh, 1970, p. 484].

The empirical argument seems rather unconvincing and the limits of the hedonistic approach are well known. The literature is too vast for a comprehensive review. The key point, however, is that taking individual preferences as given, and using them as the foundation for social allocations, may be deeply problematic from an ethical perspective and may lead to extremely undesirable conclusions. This is especially true, when agents display expensive or offensive tastes [Rawls, 1971] or when they are affected by cognitive dissonance and ex post adaptation, as in Sen’s celebrated tamed housewife paradox [Sen, 1985].

In a non-hedonistic approach, instead, the argument refers to some sort of possibly counterfactual, objective, or morally relevant, or even universal, in some relevant sense, rankings, and so the previous problems can be avoided by possibly restricting attention to ’laundered’ preferences, or preferences that agents would hold ideally or upon reflection. Then, one may argue that, once one discards epiphenomenal and morally irrelevant characteristics, agents’ true, or rational preferences are indeed largely similar, reflecting some sort of shared human nature. While adopting a non-hedonistic approach, for example, in his classic defence of utilitarianism, J.J. Smart explicitly argues that a utilitarian criterion will not lead to extreme inequalities [Smart and Williams, 1973, p.34]. Similarly, in his seminal *The Economics of Control* (1944), Abba Lerner provides an argument based on the impossibility of knowing people’s preferences. Thus “If it is impossible, on any division of income, to discover which of any two individuals has a higher marginal utility of income, the probable value of total satisfactions is maximised *by dividing income evenly*” [Lerner, 1944, p. 29, italics added]. This egalitarian implication of classical utilitarianism is endorsed by Samuelson [1964, p. 175] who argues that “if the crucial equal-ignorance assumption were really acceptable to every reasonable observer, then each and every person (subject to the postulated concave utility that renders him a risk averter) would vote for a regime of equal-distribution of income, and this constitutional feature would be instituted by unanimous vote.”

It is not entirely clear that this approach is entirely consistent with one of key formal and philosophical tenets of Utilitarianism, namely the idea that individual preferences should be taken as given and that policies should be based on individual’s actual, rather than hypothetical or counterfactual or uncertain, characteristics. Williams notes that “it is after all a well-known boast for utilitarianism that it is a realistic outlook which seeks the best in the world as it is, and takes any form of happiness into account"[Smart and Williams, 1973, p. 104] and also that "modern utilitarianism is supposed to be a system neutral beyond the preferences that people actually have. To legislate them out is not to pursue people's happiness but to remodel the world towards forms of 'happiness' more amendable to utilitarian ways of thought" [Smart and Williams, 1973, p. 131]. This reflects a commitment to the respect of individual autonomy and also, indirectly, to the equal worth of people. Yet, in principle, the ideal, non-hedonistic account can provide a reply to the criticisms based on objectionable preferences and to some versions of the inequality generation critique.

The goal of this paper is to analyse the egalitarian implications of a Utilitarian distribution, in a dynamic context when preferences are endogenously changing; and more specifically depend on past consumption habits. Habit formation and habit persistence have been studied widely from different viewpoints, including consumption theory and international economics, but to our knowledge, it has not been discussed in terms of its relevance to welfare economics and normative analysis. Habits provide a dynamic perspective which is fundamental for the evaluation of object of our analysis, the model's preference formation implies that agents who have been richer in the past (e.g. they come from rich families) are better "utility machines" (both level-wise and at the margin).

The motivation for this research question is two fold. The first question that we seek to answer is whether Bentham's positive views about *equalisation* are consistent with his normative approach on a just distribution. The second, has to do with the fact that, as we briefly discussed above, different utilities lead in general to inequality of utility and resources under a Utilitarian distribution, while the Utilitarian distribution is in general egalitarian if we consider the same utilities among individuals. This raises the question of whether under reasonable assumptions allowing for endogenous preferences, the inegalitarian implications of the Utilitarian distribution of a one period model, will diminish or even disappear in the long run.[2]

We show that consumption habit formation has egalitarian effects in a Utilitarian distribution. These results demonstrate that if we allow for preferences to depend on past consumption, the key critiques of Utilitarianism do not hold and also that Clas-

[2]The changing preferences of people has also been the focus of Marx's (short) critique to Bentham's Utilitarianism. In a endnote of chapter 24 of the 1st volume of Capital, Marx criticised Bentham, arguing that the latter's approach did not take into account that people's behaviour, and human nature in general is dependent on historical factors.

sical Utilitarianism is consistent with Bentham's views on equality. We study the distributional implications of a Utilitarian distribution in different contexts, both regarding the assumptions made about the habit formation and the time horizon of the planner's maximisation. Regarding the time horizon of the planner, we consider the following setups: (i) a single period maximisation problem, (ii) a dynamic problem where the planner maximises in every period and (iii) the intertemporal problem, where the planner is forward looking and maximises over time. Analysing the problem in different setups allows us to start from very general assumptions regarding the utility function and the habit formation in (i); and move towards the mosre standard and restrictive assumptions used in the literature in (ii) and (iii). Considering the habit formation, we study the two forms of habits used in economics literature, namely subtractive and multiplicative.

Interestingly, we find that in even though in all cases relative inequality diminishes this does not always happen in the same way. In general we find that if the habits take the subtractive form, then the distribution is such that it leads to equality by decreasing the consumption of the high habit individuals and increasing the consumption of the lower habit ones. In the case where the habits take the multiplicative form, assuming isoelastic preferences, we show that although the distribution leads to consumption equality, this may happen in a cyclical way, where in every period the difference in consumption changes signs and the rich become poor and vice versa. This effect takes place only in discrete time when the he elasticity of intertemporal substitution of consumption does not take very low values. The intuition is that when the elasticity is sufficiently high, then the planner needs to (over)compensate (relative to the next period) the individuals with low habits and thus punish the ones with high habits. This effect disappears when the time is continuous.

The structure of the rest of the paper is as follows. Section 2 discusses the relevant literature and the relation to the current paper. Section 3 introduces the assumptions of the model, the different kinds of habits' formation and presents the results of a one period case. Section 4 considers the dynamic environment where the planner is myopic and section 4 considers the forward looking equivalent. Finally, section 5 concludes and suggests further research directions.

1.2 Relevant Literature

The question which is posed in this paper touches upon a number of fields ranging from social choice theory and welfare economics to formal political theory and political philosophy. In this section we provide a brief review of some of the key contributions related to Utilitarianism and Egalitarianism on one hand and preference and habit formation on the other.

Considering the literature relating to critical approaches to Classical Utilitarianism and their relation to Egalitarianism, probably the most famous critique comes from Rawls [1971] who argued that a social planner should aim not to maximise the sum of the utilities of a given population but to maximise a bundle (of what he called) “primary” goods of the least well off individual(s) (difference principle). This approach leads to a tendency of equalisation of the “primary” goods across individuals. Sen [1980, 1985] also criticised the Classical Utilitarian distribution from an Egalitarian perspective but argued that Rawls’ use of primary goods was not the most appropriate for an egalitarian distribution and instead argued for the use of what he called “capability”. The reason for this is that if the primary goods were equally distributed, then the Rawlsian argument would be for a redistribution of income but for Sen, income should be distributed not equally but depending on peoples’ physical or mental capabilities, as a less (physically or mentally) "capable" person would need more income higher income than a more "capable" person.

Myerson [1981], analyzed the relationship between Utilitarianism and Egalitarianism with respect to the “timing effect”. The “timing effect" refers to the fact that different conclusions about a specific choice can be reached, depending on when these policies are evaluated. Myerson shows that under reasonable conditions a social choice can be either Utilitarian or Egalitarian and not both at the same time.
Although the present paper, at least to our knowledge, constitutes a novel contribution from a normative point of view, habit formation is a topic broadly discussed in economics research[3]. One of the first economists who highlighted the importance of habits in preference formation was Irving Fisher [1930] in his magnum opus, *The Theory of Interest*, where he used the notion of habits to discuss the relationship between standards of living and impatience.

Duesenberry [1949] was the first to introduce the idea that consumption habits of the previous period, affect the preferences of present consumption. From a similar point of view, Jr and Heal [1973] and Boyer [1978] formally introduced consumption habits in the neoclassical growth model and studied the optimal growth paths. Becker and Murphy [1988]- building on the Stigler and Becker [1977] model of the effects of addiction, habitual behavior, advertising, and fashions on the stability of tastes- present a forward looking maximisation model with rational addictions which provides explanations regarding empirical issues relating to consumption.

[3]See Messinis [1999].

Habit formation has been used as a tool for explaining the empirical “puzzles” in consumption theory and in (the consumption- based) asset pricing theory. Ferson and Constantinides [1991], included consumption habits and provided an explanation of the fact that consumption does not change sufficiently as a response to unexpected changes in income (excess smoothness). [4] Constantinides [1990] argues that habit persistence provides a solution to the equity premium puzzle [5] and finds empirical evidence that the consumption habit persists for a period longer than one year. Campbell and Cochrane [1995] argue that by adding a habit in the utility function, we can explain the pro-cyclical variation of the stock prices.

1.3 Economic Environment

Consider an economy with two individuals indexed by $i = 1, 2$ and a cake of size equal to 1, where the utilitarian social planner chooses how to divide the cake between the two individuals such that the total Utility is maximised. The utility of each of the individuals depends not only on present consumption levels but also on past consumption levels. This dependence on past consumption takes the form of what is referred to as “habit” in the economic literature. We first focus in a one period case where the the individuals have given habits and the planner chooses their consumption levels. This simple case can provide useful intuition and some more general results.

1.4 Single Period Problem

Let the utility function of each individual take the form

$$U = u(S(c, h)) \tag{1.1}$$

where c is the consumption level of the individual, $S(c, h)$ captures the relationship between consumption and habits that enters the utility function and can be seen as an intermediate production function.

Assumption 1: u is increasing and (quasi-) concave in S.

The assumptions regarding u and S aim to capture several psychological situations related to habit formation, namely the notions of tolerance, reinforcement and withdrawal. For our present framework, we are only interested in the fist notions and

[4] Also see Deaton Paradox [Deaton, 1992].

[5] The role of habits in providing an explanation for the equity premium puzzle has been discussed by Abel [1990, 1998]

the conditions that are imposed because of this to u and S. Becker and Murphy [1988] define tolerance (of a harmful good) as follows:

> Tolerance means that given levels of consumption are less satisfying when past consumption has been greater... [H]armful addictions ... imply a form of tolerance because higher past consumption of the harmful good, lowers the present utility from the same consumption level. (p. 682)

The economics literature, has considered two types of cases for function S which capture tolerance, namely *subtractive* and *multiplicative* forms of habits.[6] These can be expressed by the following:

Assumption S: $S(c,h) = c - \alpha h$, where $\alpha \in (0,1)$.

Assumption M: $S(c,h) = \frac{c}{h^{\gamma}}$, with $\gamma \in (0,1)$.[7]

It is easy to prove that for both of these cases the assumption of tolerance holds.[8] Note that in both cases a marginal change in h has a relatively smaller effect than a marginal change in c, by the assumption that current habits have a smaller effect in the overall utility than the current levels of consumption, i.e. $\alpha, \gamma \in (0,1)$. This can be formally expressed in the general case by:

Without loss of generality let the habit of the first individual be $h_1 = h \in \left(\frac{1}{2}, 1\right)$ and the habit of the second one be $h_2 = 1 - h$. The utilitarian social planner

$$\max_{c^1,c^2}\{u(S(c^1,h^1)) + u(S(c^2,h^2))\}, \quad (MP\ 1)$$

such that

$$c^1 + c^2 = 1. \tag{1.2}$$

Proposition 1. *Given A.1, at the solution of the maximisation problem,* $c^1 - c^2 < h^1 - h^2$ *for either AS or AM.*

Proof. At the solution of the problem,

$$u_S(S(c^1,h^1))S_{c^1}(c^1,h^1) = u_S(S(c^2,h^2))S_{c^2}(c^2,h^2). \tag{1.3}$$

[6] For more details about the differences between the two forms see Carroll [2000], Constantinides [1990], Abel [1990] and Gali [1994].

[7] Note that under both AS and under AM, u is increasing and concave with respect to consumption; see Carroll [2000].

[8] For the assumption of reinforcement to hold, we need to make some further assumptions about the utility function.

Given that $h^1 > h^2$, for either AS or AM

$$S_{c^1}(c^1, h^1) \leq S_{c^2}(c^2, h^2), \tag{1.4}$$

for all $c^1, c^2 \in (0, 1)$, and thus

$$u_S(S(c^1, h^1)) \geq u_S(S(c^2, h^2)), \tag{1.5}$$

Then from A.1, we get that

$$S(c^1, h^1) \leq S(c^2, h^2). \tag{1.6}$$

Then given that $h^1 > h^2$, we get that (1.6) holds if and only if c^1 is not sufficiently large compered to c^2, such that its positive effect on $S(.)$ is greater than the relative to h^2 negative effect of h^1 to $S(.)$. In this way (1.6) holds iff

$$c^1 - c^2 < h^1 - h^2.$$

□

In an one period maximisation, the habit stock h, can be seen as the consumption of the previous period. Hence the result shows that inequality does not grow in the same direction as past inequality. This result raises two further questions.

(i) Whether the consumption of the high habit individual is higher than the consumption of the one with lower habit ($c^1 > c^2$).

(ii) If not, whether the absolute consumption inequality is lower than inequality of habit stocks ($|c^1 - c^2| < h^1 - h^2$).

Given the generality of the functions u and S, it is not possible to answer these questions without considering the cases AS and AM separately.

Corollary 1. *Under AS, at the solution of MP 1;* $c^1 > c^2$*.*

Proof. Assuming AS, gives

$$S_{c^1}(c^1, h^1) = S_{c^2}(c^2, h^2),$$

which following the steps of the proof of Proposition 1 leads to

$$S(c^1, h^1) = S(c^2, h^2). \tag{1.7}$$

If we substitute S, we get

$$c^1 - \alpha h^1 = c^2 - \alpha h^2,$$

or

$$c^1 - c^2 = \alpha(h^1 - h^2) > 0. \tag{1.8}$$

$\square$

Under AM, it is not possible to draw conclusions about the Utilitarian distribution under the same assumptions, thus we have to make further assumptions considering the utility function. In the following, we consider the usual case in the literature of isoelastic utility[9]:

$$u(S) = \frac{S^{1-R}}{1-R}, \tag{1.9}$$

where $R > 0$.

Proposition 2. *Under AM, for any $u(S)$ given by (1.9): (i) $c^1 > c^2$ iff $R > 1$ and (ii) for $R < 1$, $c^1 < c^2$; with $|c^1 - c^2| < h^1 - h^2$ in both cases.*

Proof. (i) Similarly to the previous case, (1.3) becomes

$$\left(\frac{c^1}{(h^1)^\gamma}\right)^{-R} \frac{1}{(h^1)^\gamma} = \left(\frac{c^2}{(h^2)^\gamma}\right)^{-R} \frac{1}{(h^2)^\gamma}, \tag{1.10}$$

or

$$\left(\frac{c^1}{c^2}\right)^{-R} = \left(\frac{h^1}{h^2}\right)^{-\gamma R} \left(\frac{h^1}{h^2}\right)^{\gamma},$$

or

$$\left(\frac{c^1}{c^2}\right)^{R} = \left(\frac{h^1}{h^2}\right)^{\gamma(R-1)}, \tag{1.11}$$

which means that given that $h^1 > h^2$, $c^1 > c^2$ iff $R > 1$. Note that for $c^1 - c^2 < h^1 - h^2$, it is sufficient to show that

$$R > \gamma(R-1),$$

or

$$\frac{R}{R-1} > \gamma,$$

which is true as $\gamma < 1$ and $\frac{R}{R-1} > 1$.

[9]See Carroll [2000].

(ii) If $R < 1$, then for $|c^1 - c^2| < h^1 - h^2$ to hold, it is sufficient to prove that

$$\frac{h^1}{h^2} > \left(\frac{h^1}{h^2}\right)^{\gamma(1-R)}, \tag{1.12}$$

which is true given that $h^1 > h^2$ and $\gamma(1-R) < 1$.

□

Proposition 2 shows that in the single period problem the Utilitarian distribution is egalitarian in the sense of reducing consumption inequalities. Interestingly under AM, when $R < 1$ (high elasticity), the planner distributes more of the consumption good to the individual with the high habit stock. In this way, the Utilitarian distribution diminishes inequality which is in accordance with Bentham's positive views. On the other hand the liberal critique to Utilitarianism still holds, as individuals with high consumption habits will consume more than the individuals with low consumption habits.

1.5 Myopic Problem

In order to be able to draw conclusions on the evolution of consumption inequality, we need to make assumptions on how the habit stock evolves. Thus, assume that for every period t, the habit stock of $i = 1, 2$ is given by

$$h^i_{t+1} = \rho c^i_t + (1-\rho) h^i_t, \tag{1.13}$$

where $\rho \in (0, 1)$ captures the 'speed of adjustment' of the habits to previous consumption relative to previous habit levels. Assume for simplicity that $h^1_0 + h^2_0 = 1$. Then, given constraint (1.2) for all t:

$$h^1_t + h^2_t = 1. \tag{1.14}$$

The problem of the planner is for every t to

$$\max_{c_t} \{u(S(c_t, h_t)) + u(S(1 - c_t, 1 - h_t))\}, \quad (MP\ 2)$$

subject to (1.13).

Proposition 3. *Under AS, with u concave with respect to S, at the solution of MP 2,* $c^1 = c^2 = \frac{1}{2}$ *is the unique equilibrium and is asymptotically stable.*

Proof. From (1.8), at the solution of MP 2, for all t:

$$c_t = \frac{1}{2}[1 + \alpha(2h_t - 1)]. \tag{1.15}$$

Then (1.13) becomes

$$h_{t+1} = \rho\frac{1}{2}[1 + \alpha(2h_t - 1)] + (1 - \rho)h_t,$$

or

$$h_{t+1} = \frac{(1-\alpha)\rho}{2} + (1 - \rho + \alpha\rho)h_t, \tag{1.16}$$

from which we get that the stationary state value $h_{t+1} = h_t = \frac{1}{2}$. The eigenvalue is then equal to $1 + \rho(\alpha - 1) < 1$; thus the stationary equilibrium is asymptotically stable.

$\square$

This result highlights the long run egalitarian implications of a utilitarian distribution, under AS. As it was already clear in the previous section we need to have an explicit form for the utility function of individuals in order to be able to characterise the solution of the problem under AM.

Proposition 4. *Under AM, with u given by (1.9), at the solution of MP 2 (i) $c = \frac{1}{2}$ is the only stationary solution, (ii) it is globally stable and (iii) for $R < 1$, for all t, $sign(c^1_t - c^2_t) \neq sign(c^1_{t+1} - c^2_{t+1})$.*

Proof. (i) At the stationary state equilibrium for $i = 1, 2$, $h^i_t = h^i_{t+1}$. Then from (1.13), we get that $h^i_t = c^i_t = c$. The solution of MP 2 gives

$$c^{-R}(h^{-\gamma})^{1-R} = (1 - c)^{-R}[(1 - h)^{-\gamma})]^{1-R}. \tag{1.17}$$

From which we get, that the stationary state equilibrium is $c = \frac{1}{2}$.

(ii) From Proposition 2, we know that for $h^1 > h^2$ and $R > 1$, $c^1 > c^2$. Also note that from (1.13) we have that for all t and $i = 1, 2$, h^i_t is an increasing function of c^i_t. Hence, from the uniqueness of the stationary equilibrium we have that:

$$\lim_{t\to\infty} c^i_t = \frac{1}{2},$$

which means that the stationary equilibrium is globally stable.

Similarly, for $R < 1$, $c^1 < c^2$ with $|c^1 - c^2| < h^1 - h^2$. Given (1.13), we have that for $i = 1, 2$:

$$\lim_{t\to\infty} c_t^i = \frac{1}{2} \tag{1.18}$$

(iii) This follows from the above.

□

It is interesting to note that even though in both cases of habit formation long run consumption inequality tends to zero, this happens in a monotonic way only under AS. Under AM, if the elasticity of intertemporal substitution of consumption is relatively high, consumption converges to equality in a cyclical way, where the position of relatively rich and poor changes in every period.

1.6 Forward Looking Problem

We now consider the case where the Social planner is forward looking and maximises the discounted sum of the two individuals' utilities over time. The habit follows the same adaptive process as before but in continuous time. Thus the change of habit will be given by

$$\dot{h} = \rho(c - h) \tag{1.19}$$

where $\dot{h}$ denotes the time derivative of $h_t = h$ and $c_t = c$. For simplicity, let the habit stock of the first individual be h and of the second one $1 - h$ and let h_0 given with $h_0 \in \left(\frac{1}{2}, 1\right)$. Then, the maximisation program reads:

$$\max_c \int_0^\infty e^{-\theta t} u[S(c,h)] + u[S(1-c, 1-h)]dt, \quad (MP\ 4) \tag{1.20}$$

subject to (1.19) and the transversality conditions; given h_0 and $c,\ h \in (0,1)$, where $\theta \in (0,1)$ is the time preference. The current value Hamiltonian is

$$\hat{H} = u[S(c,h)] + u[S(1-c,1-h)] + \mu[\rho(c-h)]. \tag{1.21}$$

The necessary conditions for optimality are:

$$\hat{H}_c = 0, \tag{1.22}$$

$$\dot{\mu} = \theta\mu - \hat{H}_h, \tag{1.23}$$

and (1.19).

In the following we assume that u is given by (1.9) and also that $R < 1$, which guarantees that the utility is concave in the dynamic framework.

Proposition 5. *At the solution of MP 4, (i) the egalitarian distribution $c = h = \frac{1}{2}$ is the unique stationary state equilibrium and (ii) the equilibrium is asymptotically stable both under AS and AM.*

Proof. We consider each of the cases separately.
Under AS:

(i) Note that

$$\hat{H}_c = (c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R} + \mu\rho,$$

and

$$\hat{H}_h = -\alpha(c - \alpha h)^{-R} + \alpha[1 - c - \alpha(1 - h)]^{-R} - \mu\rho.$$

The optimality conditions become

$$(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R} + \mu\rho = 0 \tag{1.24}$$

$$\dot{\mu} = \theta\mu + \alpha\{(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R}\} + \mu\rho \tag{1.25}$$

and (1.19). Then from (1.24) we can get

$$\mu = \frac{1}{\rho}\{-(c - \alpha h)^{-R} + [1 - c - \alpha(1 - h)]^{-R}\} \tag{1.26}$$

and also

$$\dot{\mu} = \theta\mu + (\alpha - 1)\{(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R}\},$$

or

$$\dot{\mu} = (\alpha - 1 - \frac{\theta}{\rho})\{(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R}\}. \tag{1.27}$$

Also from (1.24), if we take the time derivative and substitute from (1.9) and (1.25), we get

$$-R(c-\alpha h)^{-R-1}(\dot{c}-\alpha\rho(c-h))-R[1-c-\alpha(1-h)]^{-R-1}(\dot{c}-\alpha\rho(c-h))+\dot{\mu}\rho = 0,$$

substituting from (1.27), gives

$$(\dot{c} - \alpha\rho(c - h))\{(c - \alpha h)^{-R-1} + [1 - c - \alpha(1 - h)]^{-R-1}\} =$$

$$= \frac{\rho\alpha - \rho - \theta}{R}\{(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R}\},$$

which simplifies to

$$\dot{c} = \alpha\rho(c - h) + \frac{\rho - \theta - \rho\alpha}{R} \frac{(c - \alpha h)^{-R} - [1 - c - \alpha(1 - h)]^{-R}}{(c - \alpha h)^{-R-1} + [1 - c - \alpha(1 - h)]^{-R-1}}, \quad (1.28)$$

which along with (1.19), give the 2D system which describes the behaviour of the solutions of the problem. From (1.19), we get that at the stationary equilibrium $c = h$; and from (1.28) we get that at stationary equilibrium $c = h = \frac{1}{2}$.

(ii) The Jacobian of the system at the stationary equilibrium $(\frac{1}{2}, \frac{1}{2})$ is

$$J = \begin{pmatrix} 0 & 0 \\ \rho & -\rho \end{pmatrix}.$$

Notice that $Tr(J) = -\rho < 0$. The Discriminant Δ is

$$\Delta = (Tr(J))^2 - 4Det(J) = \rho^2 > 0. \quad (1.29)$$

We know from the Routh- Hurwitz theorem that the fixed point is locally stable.

Under AM:

(i) The current value Hamiltonian reads:

$$\hat{H} = \frac{1}{1 - R}\left[\left(\frac{c}{h^\gamma}\right)^{1-R} + \left(\frac{1 - c}{(1 - h)^\gamma}\right)^{1-R}\right] + \mu[\rho(c - h)] \quad (1.30)$$

$$\hat{H}_c = -(1 - c)^{-R}[(1 - h)^{-\gamma})]^{1-R} + c^{-R}(h^{-\gamma})^{1-R} + \mu\rho, \quad (1.31)$$

and

$$\hat{H}_h = \gamma\left[\frac{(1 - c)^{1-R}\left((1 - h)^{-\gamma}\right)^{1-R}}{1 - h} - \frac{(ch^{-\gamma})^{1-R}}{h}\right] - \mu\rho. \quad (1.32)$$

So, from (1.22) we get that

$$\mu\rho = (1 - c)^{-R}[(1 - h)^{-\gamma})]^{1-R} - c^{-R}(h^{-\gamma})^{1-R}, \quad (1.33)$$

or equivalently

$$\mu = \frac{1}{\rho}\left[(1-c)^{-R}[(1-h)^{-\gamma})]^{1-R} - c^{-R}(h^{-\gamma})^{1-R}\right]. \tag{1.34}$$

From the above, we get

$$\dot{c} = -\frac{c(c-1)(A-B)}{\Gamma}, \tag{1.35}$$

where

$$A = (1-c)^R(h-1)h^{\gamma(R-1)}(h\theta + (h + (c + 2h(R-1) - 2cR)\gamma)\rho),$$

$$B = hc^R(1-h)^{\gamma(R-1)}((1-h)\theta + (h - 1 + (1 + c + 2h(R-1) - 2cR)\gamma)\rho),$$

and

$$\Gamma = hR\left(c^{R+1}(1-h)^{1+(R-1)\gamma} + (1-c)^R(c-1)(h-1)h^{\gamma(R-1)}\right).$$

Equations (1.35) and (1.19) describe the solutions of the maximisation problem.

From (1.19) we have that for $\rho \neq 0$, $\dot{h} = 0$, iff $c = h$. Note that for $c = h \neq 0$, $\dot{c} = 0$ iff $A = B$. For $c = h$ we have:

$$A = -(1-c)^{R+1}c^{\gamma(R-1)+1}(\theta + \rho - \gamma\rho),$$

and

$$B = -c^{R+1}(1-c)^{\gamma(R-1)+1}(\theta + \rho - \gamma\rho).$$

Thus $A = B$ iff

$$(1-c)^{R+1}c^{\gamma(R-1)+1} = c^{R+1}(1-c)^{\gamma(R-1)+1},$$

or

$$(1-c)^{R-\gamma(R-1)} = c^{R-\gamma(R-1)},$$

which holds for $c = \frac{1}{2}$

(ii) The elements of the Jacobian at the fixed point $c = h = \frac{1}{2}$, are

$$J_{11} = \theta + \rho\left(\gamma - \frac{\gamma}{R} + 1\right), \tag{1.36}$$

$$J_{12} = \frac{\gamma\{\theta(1-R) + \rho[2-\gamma+R(\gamma-3)]\}}{R}, \tag{1.37}$$

$$J_{21} = \rho, \tag{1.38}$$

$$J_{22} = -\rho. \tag{1.39}$$

The Trace, $Tr(J)$ and the Determinant $Det(J)$ of the Jacobian are

$$Tr(J) = \theta + \frac{\gamma\rho(R-1)}{R}, \tag{1.40}$$

and

$$Det(J) = \frac{\rho[R(\gamma-1)-\gamma](\theta+\rho-\gamma\rho)}{R}. \tag{1.41}$$

The Discriminant Δ is

$$\Delta = (Tr(J))^2 - 4Det(J) \tag{1.42}$$

Note that given $\gamma > 0$, $Det(J) < 0$ for all values of θ, ρ and R. In this case we know that $\Delta > 0$ (both eigenvalues are real numbers). Then from the Routh-Hurwitz theorem that the fixed point is a saddle point. Thus, the planner will place c_0 on the stable manifold as this is the only possible choice given the restrictions put by the transversality conditions.

□

Notice that in contrast with the results of the previous section, the distribution of the social planner takes the same form in under both AS and AM. The key difference lies on the fact that even though in both the myopic and dynamic cases under AM, the stationary solution was stable, in the former case the distribution tends to the egalitarian one in a cyclical way. The fact that this changes in the forward looking case is intuitive as the cyclical distribution, leads to a loss in total utility in the long run.

1.7 Conclusion

In this paper we have investigated how compatible is a classical utilitarian distribution in presence of endogenous preferences influenced by habits with an egalitarian conception of distributive justice. The inclusion of habit formation in the preferences has been introduced in two different ways such that habits take both the subtractive form and the multiplicative form. The effects of the distribution have been studied in three cases considering time, namely, a one period case, a myopic case and a forward looking case. This has not only helped the intuition behind the results in the forward looking case but also has allowed for more general results in the one period and myopic cases respectively.

In the general case where we studied a one period problem, we were able to show that in the solution of the maximisation problem, relative inequality is not growing at least in the same direction as the initial inequality (Proposition 1). In order to be able to have more concrete conclusions about the inequality dynamics, we needed to make more assumptions about the form of habits. In the case of subtractive form, we have shown (Corollary 1) that for every concave utility function, inequality diminishes and that the individual who consumes more, is the individual with the high habit. In the case of multiplicative form, we assumed that the utility is isoelastic. Here, we showed that in all cases, absolute inequality diminishes but if the elasticity is positive, the planner distributes more of the consumption good to the individual with the low habit stock (Proposition 2).

The results were extended in a dynamic economy, where we fist considered a myopic Social planner and then a forward looking one. In the myopic case we found that the distribution will lead to equality in the long run, in both cases of habit formation (Propositions 3 and 4). The difference between the two cases is that in the multiplicative habit form, if the elasticity is positive the stationary state solution is a spiral node. In the forward looking case we showed that for both cases the stationary solution is the egalitarian one (Propositions 5-7). We found that although in the continuous time problem and in the subtractive form of the discrete one, the solution is always asymptotically stable and the consumption of the poor grows monotonically, in the discrete case with multiplicative habits this is not always true. In the latter case there is the possibility for the stationary state to be a spiral node (Corollary 2).

Finally, we considered an economy where individuals live for two periods and their utility depends positively on the consumption of both periods. This approach is different from the usual ones in the economics literatures which discuss habit formation but it is important from a normative perspective. In this case we consider a Cobb Douglas utility and show that if the function is homogeneous of degree one, then there are infinite solutions, where in the case of being homogeneous of degree less than one, the only solution is the egalitarian one (Proposition 8). The first result, thus shows that the Utilitarian distribution can be egalitarian but this is not necessarily the case.

The contribution of this paper lies on three different levels. On a first level the relaxation of the exogenous preferences assumption and the introduction of habits is both normatively relevant and provides an extension of the analytic frame of distributive justice. On a second level, we provide a complete characterisation of the optimal paths of the dynamic optimisation programme for different forms of habits. On a third level, the results of the analysis provide a novel defence of Classical Utilitarianism in relation to its distributional consequences.

Regarding further research relating to the present paper, we can identify three straightforward areas for possible extensions that come out of the analysis up to now. The first has to do with the existence of uncertainty in the distribution of the Social planner, such that in every period a part ε of the social good is distributed randomly. The question here is whether this has an effect to our findings. The second issue is for the introduction of growth, such that the Social planner, chooses not only how to distribute the cake but also how much to invest in the next period. Finally, future research can deal with other welfare- based theories, such as a utility-based version of Rawls' maximin and Equality of Opportunity [Roemer, 1998], in a dynamic context with endogenous preference formation and possibly production.

1.8 Appendix

In this appendix we consider the case of an economy where individuals live and consume for two periods and we show that there are cases where the Utilitarian distribution is not egalitarian. Each individual has one parent and one child and her current consumption preferences depend on the consumption of the previous period. For simplicity we assume that consumption takes place within a household and the consumption of the parent and child within the same household is indistinguishable. In this case we allow the family background of an individual to influence her ability to enjoy current consumption both in levels and at the margin. All individuals have identical preferences which can be represented by a utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$, twice differentiable with $u'_1 > 0$, $u'_2 > 0$ and concave, where u'_i is the first partial derivative with respect to the variable i and u''_{jk} is the second partial derivative with respect first to j and then to k. The preferences are described by:

$$u(c^i_{t-1}, c^i_t)$$

As in the previous cases, we consider a problem where each period the Utilitarian social planner divides a cake of size one such that at every point in time $c^1_t = 1 - c^2_t$

$$V(c_0) = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t, c_{t+1}) + u(1 - c_t, 1 - c_{t+1})\right] \quad (MP\ 5)$$

Proposition 6. *Let u be concave and homogeneous of degree one, then at the solution of the maximisation problem: (i) $c_t = c_0$ for all t; and (ii) $V(c^1_0) = \frac{u(1,1)}{1-\beta}$*

Proof. (i) The Euler equation reads:

$$u'_2(c_{t-1}, c_t) - u'_2(1 - c_{t-1}, 1 - c_t) = -\beta \left[u'_1(c_t, c_{t+1}) - u'_1(1 - c_t, 1 - c_{t+1})\right] \quad (1.43)$$

Since u is homogeneous of degree one, $\forall\ x, y$, $u'_j(x, y) = u'_j(x/y, 1)$, $j = 1, 2$. Thus $c^1_{t+1} = c^1_t$, satisfies the Euler Equation and Transversality Conditions for all t.

(ii) The second part of the proposition follows from part 1 of the proof, by noting that,

$$V(c_0) = \frac{u(c_0, c_0) + u(1 - c_0, 1 - c_0)}{1 - \beta} = \frac{c_0 u(1, 1) + (1 - c_0)u(1, 1)}{1 - \beta} \quad (1.44)$$

□

Definition 1. *Let ψ: [0,1] $\to$ [0,1] be a function such that $u'_2(c^1, \psi(c^1)) \equiv u'_2(1 -$*

$c^1, 1-\psi(c^1))$ *and* $u_1'(c^1, \psi(c^1)) \equiv u_1'(1-c^1, 1-\psi(c^1))$. *Then* ψ *is defined as an intratemporal policy (IP).*

Proposition 7. *Assume that there is no IP, then: (i)* $c = \frac{1}{2}$ *is a solution of MP 5; but (ii) it is not stable.*

Proof. (i) (1.43) and the concavity of u gives that $c_t^1 = 1 - c_t^1$ for all t.

(ii) We linearise (1.43) around $\bar{c} = \frac{1}{2}$. Using the implicit function theorem we will get

$$u_{21}''(\bar{c},\bar{c})[c_t^1 - \bar{c}] + u_{22}''(\bar{c},\bar{c})[c_{t+1}^1 - \bar{c}] = -\beta\left[u_{11}''(\bar{c},\bar{c})[c_{t+1}^1 - \bar{c}] + u_{12}''(\bar{c},\bar{c})[c_{t+2}^1 - \bar{c}]\right]$$

or equivalently

$$(c_{t+2}^1 - \bar{c}) + \frac{u_{22}''(\bar{c},\bar{c}) + \beta u_{11}''(\bar{c},\bar{c})}{\beta u_{12}''(\bar{c},\bar{c})}(c_{t+1}^1 - \bar{c}) + \frac{u_{21}''(\bar{c},\bar{c})}{\beta u_{12}''(\bar{c},\bar{c})}(c_t^1 - \bar{c}) = 0$$

By Schwarz's theorem $u_{21}''(\bar{c},\bar{c}) = u_{12}''(\bar{c},\bar{c})$ and therefore it implies that the product of the two roots of the above is equal to $\frac{1}{\beta} > 1$

□

Note that if an IP which satisfies the EE exists, then the solution of the maximisation, reduces *de facto* to within- period maximisation. In the latter case, linearising around $\bar{c}$ yields

$$u_{22}''(\bar{c},\bar{c})(c_{t+1}^1 - \bar{c}) + u_{21}''(\bar{c},\bar{c})(c_t^1 - \bar{c}) = 0$$

or

$$c_{t+1}^1 - \bar{c} = -\frac{u_{21}''(\bar{c},\bar{c})}{u_{22}''(\bar{c},\bar{c})}(c_t^1 - \bar{c})$$

and if, as it seems plausible to assume that $-\frac{u_{21}''(\bar{c},\bar{c})}{u_{22}''(\bar{c},\bar{c})} \leq 1$ then the steady state is stable.

Example with Cobb Douglas Utility

Consider the case where the utility function takes the following form

$$u(c_{t-1}^i, c_t^i) = (c_{t-1}^i)^\alpha (c_t^i)^\gamma \tag{1.45}$$

where $\alpha, \gamma > 0$ with $\alpha + \gamma \leq 1$.[10]

[10]The assumption of $\alpha + \gamma \leq 1$ guarantees that the utility function is concave.

Proposition 8. *Consider MP 5, with u given by (1.45), (i) if* $\alpha + \gamma = 1$*, then all values of* $c \in (0, 1)$ *are stationary solutions while (ii) if* $\alpha + \gamma < 1$*, then* $c = \frac{1}{2}$ *is the only stationary solution.*

Proof. The Euler equation becomes

$$\gamma[(c_{t-1})^{\alpha}(c_t)^{\gamma-1} - (1-c_{t-1})^{\alpha}(1-c_t)^{\gamma-1}] = -\beta\alpha\left[(c_t)^{\alpha-1}(c_{t+1})^{\gamma} - (1-c_t)^{\alpha-1}(1-c_{t+1})^{\gamma}\right] \tag{1.46}$$

At the stationary equilibrium, $c_t = c_{t+1} = c$, thus at the stationary equilibrium the above becomes

$$\gamma[(c^{\alpha}c^{\gamma-1} - (1-c)^{\alpha}(1-c)^{\gamma-1}] = -\beta\alpha\left[c^{\alpha-1}c^{\gamma} - (1-c)^{\alpha-1}(1-c)^{\gamma}\right]$$

or

$$c^{\alpha+\gamma-1} = (1-c)^{\alpha+\gamma-1} \tag{1.47}$$

Then if:

(i) $\alpha + \gamma = 1$ then (1.47) is indeterminate.

(ii) $\alpha + \gamma < 1$ then

$$c = 1 - c \iff c + \frac{1}{2}$$

□

This result shows that the Utilitarian distribution is not necessarily egalitarian and thus provides an example where Bentham's view of equality is not necessarily consistent with the Utilitarian ethic.

Chapter 2

Unequal Societies with Equality of Opportunity

2.1 Introduction

> "An inequality is allowed only if the institution that allows it works to the greatest advantage of the least advantaged." [Rawls, 1971, p.302].

The publication of Piketty's *Capital* [Piketty, 2014] and other empirical works on the issue of rising income and wealth inequality[1] has drawn the attention of both the mainstream media and economics research. The extent of growth in inequality can be summarised by the title of a recent *Oxfam* report: "Just 8 men own same wealth as half the world" [Oxfam, 2016]. A different, but related part of recent empirical work has focused on the inequality of opportunity to education and the limits that social mobility people face due to their background.[2] The aim of this paper is to use insights from the latter literature and answer the question on whether the increasing inequality observed in the data is *fair* according to the most widely accepted ethical liberal view, namely *Equality of Opportunity* (EOp).

Since Rawls' *Theory of Justice* [Rawls, 1971], in both the philosophy and normative economics literature, the question of defining a *just* distribution has focused around the distinction between people's *circumstances* beyond one's control and the choices one makes. This *cut* was made clear by Dworkin [1981] who argued that there are two kinds of personal characteristics: the ones which are related to a person's environment and for which they should not be held responsible for, like parental

[1] For other relevant empirical works on inequality see Piketty and Zucman [2014], Saez and Zucman [2016] and references therein.

[2] For example see Chetty et al. [2016].

background, and the ones for which the person should be held responsible for. According to Dworkin, this cut was between the preferences of a person, for which they should be held responsible and their resources for which they should not be held responsible for. Thus, if we assume that there are no differences in talents (or handicaps)[3], for Dworkin the fair and responsibility- sensitive distribution is the one which allocates resources equally among individuals, even if this means inequality of welfare, which would then be due to difference in tastes. Cohen [1989], argued that even though the distinction between circumstances and choices is correct, Dworkin's cut had been misplaced, because individual preferences are also affected by their resources. Based on this, Cohen proposed that the correct *responsibility- sensitive* egalitarian policy should be aiming to equalise, not resources but opportunities for advantage[4].

Fleurbaey [1995] and Bossert [1995] proved that it is not possible for a policy to achieve both (i) full accountability for differences in outcomes[5] which stem from differences in preferences and (ii) full compensation for ability differences. Because of this issue, the economics literature has made "concessions" in at least one of (i) or (ii), leading the *responsibility- sensitive* egalitarian welfare economics, to develop in two broad directions. According to the first approach [Fleurbaey, 2008, Fleurbaey and Maniquet, 2011], differences in skills should be compensated for and individuals should be held responsible for their preferences. The second approach put forward by Roemer [1998] emphasises the fact that individuals' circumstances, also affect their preferences, and thus people should be only held partially responsible for their preferences. According to Roemer [1998], this can be overcome by dividing individuals into types according to the characteristics which are due to circumstances. Then, within a given type, individuals would differ according to the characteristics for which they can be held responsible for relatively to the other individuals in the same type. Assuming that the outcomes are affected by both types of characteristics, then the distribution of outcomes *within* a type will be due decisions that individuals could be *relatively* held responsible for, while the same is not true for the distribution of outcomes across all individuals.

Roemer [1998] argues that the EOp policy should aim to equalise (in some average sense) the achievements (or outcomes) across types but not within types. This would be achieved by dividing the individuals within a type into centiles according

[3]In the case of different individual circumstances Dworkin [1981] proposes a no envy insurance scheme.

[4]Cohen's notion of equality of opportunity for advantage is a more general case of equality of opportunity for welfare proposed by Arneson [1989].

[5]Outcomes could be levels of advantage [Cohen, 1989, Roemer, 1998], welfare, payoffs etc.

to their preferences and then maximising the minimum achievement across individuals in each centile, for each centile across types. Due to the complexity of this approach Roemer proposed a "compromise" solution, according to which the EOp policy maximises a weighted average of the minimal utilities across individuals who have the same preferences. In this way, as Fleurbaey [2008] suggests, the first approach is a middle way between outcome egalitarianism and libertarianism while the second approach is a middle way between outcome egalitarianism and utilitarianism. Along similar lines Van de gaer [1993] proposed a simpler policy, which maximises the average utility of the type for which average utility is lowest.

In order to answer the question of whether increasing inequality can be seen as fair according to EOp, we employ a dynamic model of two social classes and infinite generations. The relative income level of a generation defines what we will call socioeconomic status or simply status. Status affects both the circumstances and preferences of the next generations, through different ways which can be related to different issues such as financial resources for education and/ or inheritance of parental social capital. Given that the outcomes of one generation are affected indirectly not only by the outcomes of the previous one, but also by the outcomes of *all* the previous ones, then a *just* distribution would be the one that maximises the outcome of the worst off individuals of any point in time and if this problem has more than one solutions, the appropriate one, would be the one which maximises the outcomes of the second, third (and so on) worst off.

Our approach builds on Piketty [1995, 1998], Roemer [1996, 1998], Roemer and Veneziani [2004] and Loury [1976]. More specifically: (i) the assumptions on preferences are similar to Piketty [1995], (ii) status captures the public perception of one's skills or how 'smart' they are as in Piketty [1998], (iii) status affects the marginal return of effort as in Loury [1976] and (iv) the equilibrium concept is an extension of Roemer [1996, 1998] and Roemer and Veneziani [2004]. We study the effects of an Intergenerational EOp (IEOp) policy in two different economic environments which we call *Greenville* and *Blueville* respectively. In both places individuals choose a costly action, i.e. effort, which generates welfare gains. The difference between Greenville and Blueville is that in the former, parental background affects the marginal return of the costly action and through this the incentives of individuals, while in the latter it only affects their incentives but not the marginal return of their effort.

We find that in the first environment the IEOp policy leads to increasing inequality, in the second one inequality diminishes and in the long run disappears. In this way, we show that in the case where background can affect not only preferences but also material conditions, the EOp policy is not sufficient in reducing inequality. We also show that in Blueville where inequality diminishes, the EOp policy is the same as the Utilitarian one. In this way our results highlight the importance of the structural characteristics of an economy and at the same time raise questions about the long run egalitarian implications of the EOp ethic.

Even though EOp policies have intergenerational implications, there is very limited work on this aspect. Roemer and Veneziani [2004] have considered the effects of EOp in an intergenerational framework and have showed that EOp for some objective condition is incompatible with human development over time. Roemer and Ünveren [2017] have studied the long-term effects of policies intended to equalise opportunities among different social classes and have showed that private investment in education is a major barrier to equalising opportunities in the long run. The present paper contributes and extends this literature (i) by introducing a more general, equilibrium concept which is relevant for intergenerational policies and (ii) by showing the IEOp policies can lead to different results depending on the economic environment which these are implemented.

The present paper is related to the political theory literature, on intergenerational justice e.g. McKerlie [1989, 2001a,b, 2012], Temkin [1992, 1993], Daniels [1988, 1993, 2008], Bidadanure [2015, 2016] and Galanis and Veneziani [2017]. With the exception of Galanis and Veneziani [2017], this literature has focused on the distribution between individuals at different segments of their lives (for example young versus old) without taking into account how the distribution in one generation may have implications for the rest. Contrary to Galanis and Veneziani [2017], in the present paper we do not assume different welfare between different segments of individuals' lives but we allow for groups with different social backgrounds.

Our work also contributes to the literature on status and inequality. The origins of the literature on the effects of status are the seminal works of Rae [1834], Veblen [1922] and Duesenberry [1949] who argued that the consumption patterns of individuals are relative to the patterns of their close environment. The works of Frank [1985], Cole et al. [1992], Robson [1992], Clark and Oswald [1996], Corneo and Jeanne [1999], Moav and Neeman [2010], Becker et al. [2005] and Ray and Robson [2012] show how aiming to appear to have high status, leads to conspicuous consumption motives which can lead to persistent inequality. In the present paper, status plays a different role than in the previous literature. Here status influences the marginal return of effort which can be seen as the effect of differences in availability of financial resources and/ or as difference in social capital. In this way, our approach is closely related to the views of Coleman [1988, 1990, 1994] who have argued that social capital is key in the acquiring human capital and also determines the effectiveness of the latter. Recent works on the effects of social capital include Glaeser et al. [2002] has studied the formation of social capital and [Chou, 2006] and [Jennings and Sanchez-Pages, 2017] who have examineed the role on social capital in relation to growth and conflict respectively. In this way our paper also provides a link between the literature on social capital on one hand and on status and inequality on the other.

The structure of the rest of the paper is as follows. Section 2 presents the formal EOp programme as put forward by Roemer [1996, 1998] and extends it into an intergenerational context. Sections 3 and 4 present the results. Section 5 concludes.

2.2 Intergenerational Equality of Opportunity

In this section we define the Intergenerational EOp (IEOp) programme in a general setup. The definition of the programme proposed in the present paper is more general compared both to the static one of Roemer [1996, 1998] or the dynamic ones of Roemer and Veneziani [2004] or Roemer and Ünveren [2017]. In the following we will start by stating the EOp programme before arguing how should this be extended in the intergenerational context.

2.2.1 Equality of Opportunity programme

Consider an environment with N individuals, each of whom has several attributes that chracterise them. Based on these, any given society can divide the individuals in T different types $\mathcal{T} = \{1, 2, \ldots, T\}$ according to their circumstances; such that a type consists of all the individuals with the same circumstances.[6] Let p_k be the frequency of type k among the individuals. Individuals within the same type may have different characteristics which do not constitute circumstances, for example preferences regarding consumption over leisure time. For simplicity call this characteristic effort denoted by e. Note that more often than not, circumstances affect preferences and choices,[7] which is also the case in the present paper.

Let $u^k(x, e)$, be the welfare (or advantage) of an individual of type k, who uses amount x of a resource R allocated by the society and exerts e level of effort. Suppose that the Social Planner has a fixed amount of a resouce R to distribute

[6]Defining what constitutes a circumstance and what does not, is largely a political question related to each society in consideration.

[7]For more on this see Cohen [1989].

among the individuals. According to the EOp ethic, the Planner treats the members of each type identically. Thus R is distributed, using a policy $\psi = (\psi^1, \psi^2, \ldots, \psi^T)$, with $\psi^i : \mathcal{R}^+ \to \mathcal{R}^+$ allocation rules, for $i = 1, \ldots, T$, such that each member of a given type k will receive ψ^k.

As individuals within a type may have different preferences, assume that the effort e of the individuals within a type $k \in \mathcal{T}$, under the allocation rule ψ^k has a distribution measure $F^k_{\psi^k}$. Then, a policy ψ is feasible if the following budget constraint is respected

$$\sum_{i=1}^{T} \int \psi^i(e) dF^i_{\psi^i} \leq R.$$

Call this set policies Ψ.

Let $e^k(\pi, \psi^k)$ be the level of effort exerted by an individual at the π^{th} centile of the effort distribution within type k facing an allocation ψ^k. Based on this we can express the welfare of individuals in an indirect form depending on their type k, centile of effort level within a type π and policy ψ^k: $v^k(\pi, \psi^k)$. According to an EOp view the *desideratum* is to equalise the welfare of individuals who exert the same amount of effort across types. In this way, individuals are not held responsible for their circumstances but are held responsible for their choices. Suppose that, one is only concerned with a specific centile π. Then we can express the problem as

$$\max_{\psi \in \Psi} \min_{k \in \mathcal{T}} v^k(\pi, \psi^k).$$

The EOp policy would then require to do the same for every π. Given the difficulty of this problem; Roemer [1996, 1998] proposes a compromise according to which each centile will get an equal weight. According to this the problem is expressed as

$$\max_{\psi \in \Psi} \frac{1}{100} \sum_{\pi=1}^{100} \min_{k \in \mathcal{T}} v^k(\pi, \psi^k) \quad \text{(MP 0)}$$

The objective above gives a weight of $\frac{1}{100}$ to every centile, and in this way provides an aggregate programme for all π. Note that, the *Utilitarian* objective and the standard *Rawlsian maximin*, are just extreme cases of the EOp formulation above. In this way, using an EOp framework, Utilitarianism can be thought as the objective of a society that does not recognise any of its individuals' characteristics as circumstances. On the contrary the Rawlsian maximin, means that a society considers all the characteristics of its individuals as circumstances. Conversly, this means that ,depending on the assumptions of an economic model, the EOp objective can take the Utilitarian form (if for example all agents are identical) or the Rawlsian one (if for example there is only one different characteristic among agents and this is considered a circumstance).

2.2.2 Intergenerational Equality of Opportunity.

In the intergenerational context, the date of birth of an economic agent is clearly a circumstance. Early work on intergenerational justice[8] has used models with representative agents in each generation, which means that the only difference between agents is their date of birth. In this type of models, the policy objective is the (Rawlsian) maximin but is consistent with the EOp ethic given that there is only one characteristic which is different among individuals (date of birth) which is considered a circumstance.

Recent work on intergenerational distributive policies [Roemer and Veneziani, 2004, Roemer and Ünveren, 2017] has included heterogeneous agents with different intragenerational circumstances which have intergenerational effects. Roemer and Veneziani [2004] have taken into account both the agents' date of birth and within period differences when proposing the EOp programme, while Roemer and Ünveren [2017] focused only on within period differences and studied the "myopic" IEOp. The IEOp programme presented below, extends the policy objective of Roemer and Veneziani [2004] by taking a lexicographical priority instead of a maximin one.

Let $\boldsymbol{x} = \{x_t\}_{t=1}^{\infty}$ denote the infinite sequence of elements x_t. Then the solution of the IEOp programme is a choice of a sequence $\boldsymbol{\psi} = \{\psi_t\}_{t=1}^{\infty}$ with $\psi_t \in \Psi$, such that:

$$\max_{\boldsymbol{\psi}} \frac{1}{100} \sum_{\pi=1}^{100} \min_{t,\ k \in \mathcal{T}} v^k(\pi; \psi_t^k) \quad \text{(MP 1)}$$

Let v^{1*} be the solution of (MP 1), then for $v_t^i \neq v^{1*}$:

$$\max_{\boldsymbol{\psi}} \frac{1}{100} \sum_{\pi=1}^{100} \min_{t,\ k \in \mathcal{T}} v^k(\pi; \psi_t^k) \quad \text{(MP 2)}$$

$\vdots$

Let $v^{(I-1)*}$ be the solution of (MP I-1), then for
$v_t^i \neq v^{1*}, v^{2*}, \dots, v^{(I-1)*}$:

$$\max_{\boldsymbol{\psi}} \frac{1}{100} \sum_{\pi=1}^{100} \min_{t,\ k \in \mathcal{T}} v^k(\pi; \psi_t^k) \quad \text{(MP I)}$$

[8] For example see Arrow [1973], Dasgupta [1974] and Solow [1974].

for $I \to \infty$.

The need for this generalisation lies on the fact that there exist cases where there are more than one feasible maximin solutions and more constraints are needed. The program above is arguably the most consistent extension to the EOp ethic.

2.3 Greenville

2.3.1 Economic Environment

The basic structure of our economic environment, both in Greenville and in Blueville, is closely related to Piketty [1995]. We consider, an infinite, discrete time horizon, $t = 1, 2, \ldots$ and a continuum of individuals with mass one, where each individual has one offspring. In every period the individuals produce and consume a non storable good by exerting effort e. Each of the individuals belongs to a social class depending on their income. For simplicity call the high income social class *rich* (r) and the low income one, *poor* (p) and let α be the fraction of the population who are poor and $1 - \alpha$ the fraction of the rich ones, with $\alpha > 1/2$.

The welfare of individuals depends positively on their consumption level c and negatively on the effort e that they exert:

$$u(c, e) = c - \frac{e^2}{2}, \tag{2.1}$$

The consumption level of an individual i, is given by

$$c^i = (1 - \tau)y^i + \tau\bar{y}, \tag{2.2}$$

where τ is the tax rate, chosen according to an EOp ethic, y^i is the pre-tax income of individual i and $\bar{y}$ is the average before tax income of all individuals from both classes, such that:

$$\bar{y} = \alpha y^p + (1 - \alpha)y^r \tag{2.3}$$

In the models of Roemer and Ünveren [2017] and Piketty [1995, 1998] the income level is fixed over time but there is a possibility for individuals to change class (intergenerational mobility). Here there is no possibility for intergenerational mobility but income level is endogenous. More specifically, we assume that the marginal return to effort is higher for rich people compared to poor ones, because even though the effort cost is the same for both groups, the net benefit is different. Formally, we assume that

$$y^i(s^i, e^i) = s^i e^i, \tag{2.4}$$

where s^i which we will call *status* henceforth, captures the productivity of effort, which depends on a person's social origins (or socioeconomic status). As Loury has argued:

> There are many reasons why a child's opportunities to acquire skills vary with the economic success of her parents. For example, the quality of schooling any child receives varies considerably across communities and tends to be higher in the suburbs than in the central city. Where there is housing segregation based on income, and the quality of neighborhood schools shows a positive correlation with the community's wealth, a child's educational opportunities can be expected to vary directly with parental economic achievements. Further the absence of a perfect capital market for educational loans means that the opportunity for higher education and the quality of that education will be sensitive to an individual's socioeconomic background. [Loury, 1976, p. 155].

Given (2.4) the above can be captured by the following:

$$\frac{y_t^i}{y_t^j} = \frac{s_t^i e_t^i}{s_t^j e_t^j} = \frac{s_t^i}{s_t^j} = \frac{c_{t-1}^i}{c_{t-1}^j}. \tag{2.5}$$

The assumption between social origins and effort productivity has several possible justifications supported by recent empirical studies.[9] For example if status captures the positive effects of social networks (or social capital)[10], then effort can refer for example to hours of study during undergraduate studies. In a different context, s^i could capture the difference in quality of education, the length of education, or even a combination of the two. In these cases e could refer to the choices one makes on whether it is worth trying hard when they cannot afford to go to a top university; or whether they should do a post graduate degree or not.

Following the economics literature on status capturing the *relative position* of individuals and taking (2.5) into account we formally define status as

$$s_t^i = \frac{c_{t-1}^i}{c_{t-1}^i + c_{t-1}^j}, \tag{2.6}$$

[9] For example see Hershbein [2016] and Chetty et al. [2016].

[10] The role of social capital as effectiveness of human capital has been discussed in Coleman [1988, 1990, 1994].

with $j = p, r$ and $j \neq i$, where c_{t-1}^i is the parental after tax income of individual i. In order to simplify the notation henceforth let $s_t^p = s_t$ and $s_t^r = 1 - s_t$. We assume that in the first period the poor have lower status than the rich, i.e. $s_1 < \frac{1}{2}$ is relatively more than the population of the rich.

Optimal Effort

Consider the problem of individuals during their lifetime, where there the status is given. Individuals choose how much effort to exert in order to maximise their welfare, given the tax. Hence, each of the individuals from class $i = p, r$

$$\max_{e^i} \left\{ c^i - \frac{(e^i)^2}{2} \right\}$$

subject to (2.2) and (2.4). Thus, the optimal effort of the poor, e^p, is

$$e^p = (1 - \tau)s, \tag{2.7}$$

while for the rich it is

$$e^r = (1 - \tau)(1 - s). \tag{2.8}$$

Hence, due to differences in productivity the rich have an incentive to exert more effort than the poor. (Add empirical literature and say why this is not captured in Piketty for example) The pre-tax income of the representative individual of each class is:

$$y^p = (1 - \tau)s^2, \tag{2.9}$$

and

$$y^r = (1 - \tau)(1 - s)^2, \tag{2.10}$$

respectively. Given this, average income $\bar{y}$ can be expressed as:

$$\bar{y} = (1 - \tau)[\alpha s^2 + (1 - \alpha)(1 - s)^2] \tag{2.11}$$

In this way, given (2.2), (2.9) and (2.11) the consumption of the poor can be expressed in terms of s and τ as

$$c^p = (1 - \tau)^2 s^2 + \tau(1 - \tau)[\alpha s^2 + (1 - \alpha)(1 - s)^2]. \tag{2.12}$$

Similarly, given (2.2), (2.10) and (2.11) the consumption of the rich can be expressed in terms of s and τ as

$$c^r = (1-\tau)^2(1-s)^2 + \tau(1-\tau)[\alpha s^2 + (1-\alpha)(1-s)^2]. \tag{2.13}$$

Based on the above, the individual utilities take the indirect form

$$v^p(\tau) = \frac{(1-\tau)^2(s)^2}{2} + \tau\bar{y}, \tag{2.14}$$

and

$$v^r(\tau) = \frac{(1-\tau)^2(1-s)^2}{2} + \tau\bar{y}. \tag{2.15}$$

From the above and given (2.6), the evolution of status can be expressed as:

$$s_{t+1} = \frac{(1-\tau_t)^2 s_t^2 + \tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}{(1-\tau_t)^2[(1-s_t)^2 + s_t^2] + 2\tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}$$

or

$$s_{t+1} = \frac{s_t^2 + \tau_t(1-\alpha)(1-2s_t)}{1-2s_t+2s_t^2+\tau_t(1-2s_t)(1-2\alpha)} \tag{2.16}$$

IEOp equilibrium

Consider the following programmes:

Given s_1,

$$\max_{\boldsymbol{\tau}} \min_{t,\ i} v_t^i, \quad \text{(MP 1)}$$

subject to (2.6) for all t.

Let v^{1*} be the solution of (MP 1), then for $v_t^i \neq v^{1*}$:

$$\max_{\boldsymbol{\tau}} \min_{t,\ i} v_t^i, \quad \text{(MP 2)}$$

subject to the solution of (MP 1) and (2.6) for all t.

$\vdots$

Let $v^{(I-1)*}$ the maximin welfare at the solution of (MP I-1), then for $v_t^i \neq v^{1*}, v^{2*}, \ldots, v^{(I-1)*}$:

$$\max_{\boldsymbol{\tau}} \min_{t,\ i} v_t^i, \quad \text{(MP I)}$$

subject to the solution of (MP 1), (MP 2), ...(MP I-1) and (2.6) for all t, where $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$.

Given this, the equilibrium is defined as follows.

IEOp Equilibrium: The IEOp equilibrium is a sequence space of tax rates $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$, with $\tau_t \in [0,1]$ and effort levels $\boldsymbol{e^r, e^p}$ with $\boldsymbol{e^i} = \{e_t^i\}_{t=1}^{\infty}$, $i = p, r$ which solves (MP I), for $I \to \infty$.

Before we analyse the properties of the EOp equilibrium, we will study two special cases, which will help with the intuition for the general case: the one period problem in section 3.2; and then a dynamic myopic one in section 3.3.

Given that the EOp criterion used here is welfare based, it is crucial to know whether is is possible for the rich to be ex post worst off than the poor.

Lemma 1. *For all t, there exists no (linear) tax rate $\tau_t \in (0,1)$ such that the welfare of the poor is higher than the welfare of the rich.*

Proof. We want to show that $\forall\ \tau_t \in (0,1)$

$$v^p(\tau_t, s_t) < v^r(\tau_t, s_t), \tag{2.17}$$

or from (2.14)

$$s_t^2 < (1 - s_t)^2,$$

which is always true.

□

2.3.2 Single Period Problem

Consider a single period problem, equivalent to (MP 1) for $t = 1$. Then, given Lemma 1, the equilibrium EOp tax rate τ, solves[11]

$$\max_{\tau} \left\{ \frac{(1-\tau)^2(s)^2}{2} + \tau(1-\tau)[\alpha s^2 + (1-\alpha)(1-s)^2] \right\} \quad \text{(MP 1')}$$

Proposition 9. *Let τ^* be the tax rate at the solution of (MP 1'), then*

(i) $\tau^* = \frac{(1-\alpha)(1-2s)}{s^2+2(1-\alpha)(1-2s)}$,

(ii) τ^* *is decreasing in both α and s.*

[11] We have dropped the time subscript for simplicity.

Proof. (i) From the first order condition, the equilibrium EOp tax rate is

$$\tau^* = \frac{(1-\alpha)(1-2s)}{s^2 + 2(1-\alpha)(1-2s)}. \tag{2.18}$$

(ii) The derivative with respect to α is:

$$\tau'_\alpha = \frac{-(1-2s)[s^2 + 2(1-\alpha)(1-2s)] + 2(1-2s)(1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2}, \tag{2.19}$$

which simplifies to

$$\tau'_\alpha = \frac{-(1-2s)s^2}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

Also, the derivative with respect to s is:

$$\tau'_s = \frac{-2(1-\alpha)[s^2 + 2(1-\alpha)(1-2s)] - [2s - 4(1-\alpha)](1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2}, \tag{2.20}$$

or

$$\tau'_s = \frac{-2s(1-s)(1-\alpha)}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

□

Proposition 9, argues that *ceteris paribus* higher inequality in the previous generation and higher relative population of the rich both will lead to a higher tax rate. Taxation affects the welfare of individuals via two economic channels. On one hand high taxation means high redistribution, while on the other, high taxation has a negative effect to individuals' incentives to exert effort. For simplicity call the first channel *redistribution channel* and the second one *incentive channel*. The relative strength between these two channels, depends on the population shares between the two classes. For a given level of inequality, a high (low) proportion of poor individuals means that the benefits of the redistribution channel are relatively low (high) while the effects through the incentive channel are relatively high (low).

2.3.3 Myopic Problem

The model shows that, total redistribution, i.e. setting $\tau = 1$ is never optimal as this would affect the incentives of both the rich and the poor, who would exert no effort. In this way, in the one period problem equality is not possible. The question which follows is how relative inequality, captured by s_t, evolves in time. Prior to analysing the IEOp equilibrium, consider an *intermediate* case where the Social Planner chooses $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$ which solves the following:

$$\max_{\boldsymbol{\tau}} \min_{i} v_t^i, \quad \text{(MP M)}$$

subject to (2.6) for all t.

As discussed in the previous section, this type of problem, call it *Myopic*, is a special case of the general IEOp (forward looking) problem, as in the Myopic programme the date of birth of an individual is not considered as a circumstance.[12] Before we present the solution of (MP M), let us define the following:

Stationary Equilibrium: A stationary equilibrium is a sequence space $\{\boldsymbol{\tau}, \boldsymbol{e^r}, \boldsymbol{e^p}\}$ which solves individuals' maximisation problems and $s_t = s_1$ for $i = p, r$.

Proposition 10. *Consider (MP M), then the following are true:*

(i) There exists a unique stationary equilibrium solution $s^*(\alpha) \in (0, \frac{1}{2})$ *with* $s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha - \sqrt{4(\alpha-1)^2 + 1}\,\right]$.

(ii) For $s_1 \neq s^*(\alpha)$*, at the equilibrium solution path,* s_t *converges to* $s^*(\alpha)$*.*

Proof. From (2.16) and τ_t^* from Proposition 1, at the solution of (MP M),

$$s_{t+1} = \frac{(1-s_t)^2 - \alpha(1-2s_t)}{2s_t^2 + (3-2\alpha)(1-2s_t)}. \tag{2.21}$$

This is equivalent to

$$s_{t+1} - s_t = \frac{1}{2}\left[1 - 2s_t + \frac{2s_t - 1}{3 + 2s_t(s_t - 3) + 2\alpha(2s_t - 1)}\right]. \tag{2.22}$$

(i) At the stationary equilibrium $s_{t+1} = s_t$. Then form (2.22) we get $s_{t+1} = s_t$ for $s = \frac{1}{2}$, or for $s_t \in (0, \frac{1}{2})$, the solutions of (2.22) are the same as the solutions of

$$s^2 + s(2\alpha - 3) + 1 - \alpha = 0, \tag{2.23}$$

and these are

$$s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha \pm \sqrt{4(1-\alpha)^2 + 1}\,\right]. \tag{2.24}$$

[12] As mentioned in the previous section, this problem is the same as in Roemer and Ünveren [2017].

Given $\alpha \in (\frac{1}{2}, 1)$, $\sqrt{4(1-\alpha)^2+1} > 1$. Thus

$$\frac{1}{2}(3 - 2\alpha + \sqrt{4(1-\alpha)^2+1}) > 2 - \alpha > 1,$$

which is not possible given that $s \in (0, \frac{1}{2})$. This means that the only possible solution is

$$s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha - \sqrt{4(1-\alpha)^2+1}\,\right]. \tag{2.25}$$

(ii) Let $F(s_t) = s_{t+1} - s_t$. Note that $F(s_t) < 0$ for all $s_t \in (s^*(\alpha), \frac{1}{2})$ and $F(s_t) > 0$ for $s_t \in (0, s^*(\alpha))$ which proves the secod statement of the Proposition.

□

This shows that it is impossible for the EOp policy to lead to equality of income between the rich and the poor at any point in time in the future. If we consider a special case, where status captures educational opportunities, then our result is similar to Roemer and Ünveren [2017] where inequality is persistent when education depends only on parental status. As mentioned earlier, in Roemer and Ünveren [2017] the income between different social groups is constant, so in this way our result differs in the sense that we also show the degree of inequality in time. Also, in our model the relative population shares are important for the level of long run inequality.

Corollary 2. *At the stationary equilibrium of (MP M),* $\frac{\partial s^*(\alpha)}{\partial \alpha} < 0$.

The graphs below show the evolution of inequality captured by status, for different values of α. The vertical axis is s_{t+1}, while the horizontal is s_t.

- $\alpha = 0.9$.

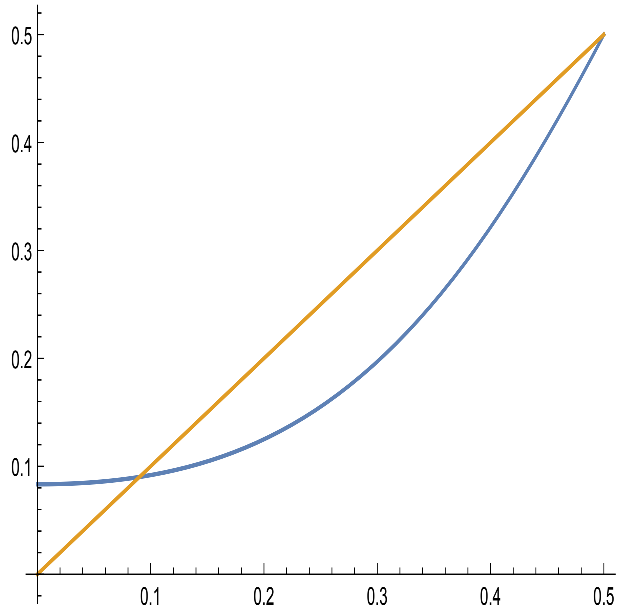


Figure 2.1: $s^*(0.9)$ close to 0.09001

- $\alpha = 0.99$.

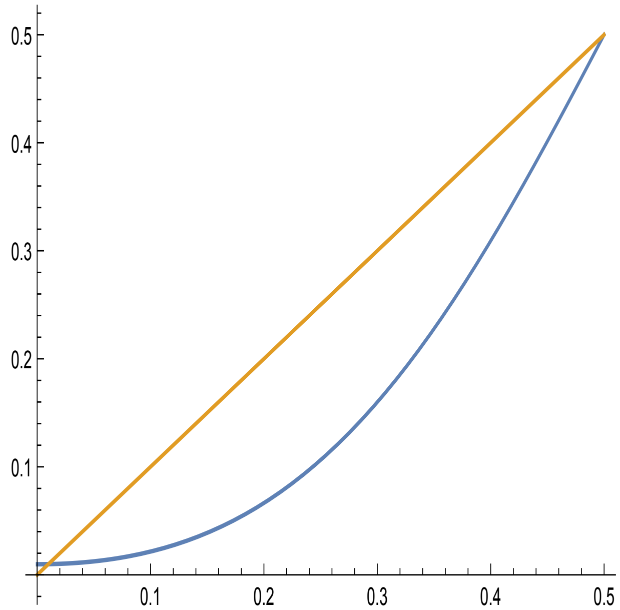


Figure 2.2: $s^*(0.99)$ close to 0.0099

- $\alpha = 0.999$.

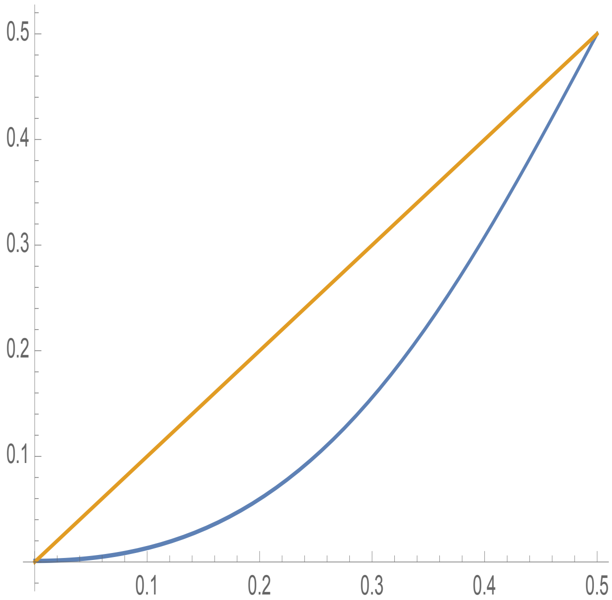


Figure 2.3: $s^*(0.999)$ close to 0.00099

The results demonstrate that, not only it may be *fair* according to a myopic EOp ethic for inequality to grow, but also that a higher relative population of the poor leads to higher long run inequality. As discussed earlier the population fractions of rich and poor individuals affect the relative importance of the redistribution channel of taxation compared to the incentive one. A high proportion of poor means that *ceteris paribus* they will contribute more to output, which in turn will increase the importance of the incentive channel of taxation and lead to lower taxes and higher inequality (captured by s).

2.3.4 Forward Looking Problem

As we have shown, there exists a trade off between redistribution and efficiency, given by the two channels through which the tax rate affects income and welfare. Let $\tilde{\tau}_t(s_t)$ be the tax rate such that $s_{t+1} = s_t$, given by

$$\tilde{\tau}_t(s_t) = \frac{s_t - s_t^2}{1 - s_t - a(1 - 2s_t)}. \tag{2.26}$$

Lemma 2. *Let $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$, then $\tilde{\tau}_t > \tau_t^*$.*

Proof.

$$\tilde{\tau}_t(s_t) - \tau_t^*(s_t) = \frac{N(s_t)}{D(s_t)},$$

where

$$N(s_t) = -1 + 2\alpha - \alpha^2 + 5s_t - 9\alpha s_t + 4\alpha^2 s_t - 8s_t^2 + 12\alpha s_t^2 - 4\alpha^2 s_t^2 + 5s_t^3 - 4\alpha s_t^3 - s_t^4,$$

and

$$D(s_t) = (1 - \alpha - s_t + 2\alpha s_t)(2 - 2\alpha - 4s_t + 4\alpha s_t + s_t^2)$$

Note that

$$1 - \alpha - s_t + 2\alpha s_t = 1 - \alpha + s_t(2\alpha - 1) > 0,$$

and

$$2 - 2\alpha - 4s_t + 4\alpha s_t + s_t^2 = 2(1-\alpha) - 4s_t(1-\alpha) + s_t^2 = (1-\alpha)(1-2s_t) + s_t^2 > 0.$$

Thus $D(s_t) > 0$ for all $s_t \in (0, \frac{1}{2})$ and $\alpha \in (\frac{1}{2}, 1)$. Hence it is sufficient to show that $N(s_t) > 0$. $N(s_t)$ can be expressed as

$$N(s_t) = (1 - 2s_t)^2(s_t - 1 + \alpha) + \frac{s_t^3(1 - s_t)}{1 - \alpha}.$$

Let $N_1 = (1 - 2s_t)^2(s_t - 1 + \alpha)$ and $N_2 = \frac{s_t^3(1-s_t)}{1-\alpha}$. Note that $N_2 > 0$, and

$$\frac{\partial N_2}{\partial s_t} = \frac{s_t^2(3 - 4s_t)}{1 - \alpha} > 0,$$

while $N_1 < 0$, for $s^*(\alpha) < s_t < 1 - \alpha$ and $N_1 > 0$ for $1 - \alpha < s_t < \frac{1}{2}$. Hence $N(s_t) > 0$ for $1 - \alpha < s_t < \frac{1}{2}$; so we need to prove that this is also the case for $s^*(\alpha) < s_t < 1 - \alpha$. Note that

$$\frac{\partial N_1}{\partial s_t} = (1 - 2s_t)(5 - 6s_t - 4\alpha),$$

which means that N_1 is increasing for $s^*(\alpha) < s_t < \frac{5-4\alpha}{6}$ and decreasing for $\frac{5-4\alpha}{6} < s_t < \frac{1}{2}$. This then means that $\frac{\partial N(s_t)}{\partial s_t} > 0$ for $s^*(\alpha) < s_t < \frac{5-4\alpha}{6}$. Given that $\frac{5-4\alpha}{6} > 1 - \alpha$, in order to conclude the proof, it is sufficient to show that $N\left(s^*(\alpha)\right) \geq 0$, which is true as $N\left(s^*(\alpha)\right) = 0$.

□

This result states that for any level of inequality lower than $s^*(\alpha)$, there is a trade off between maximising the welfare of the least well off in a given generation; and keeping at least the same levels of inequality (or status) for the next.

Proposition 11. *Let* $s_1 \in \left[s^*(\alpha), \frac{1}{2}\right)$. *At the IEOp equilibrium,* $v^p(s_t, \tau_t)$ *is constant over* t.

Proof. We will prove this by induction.

(i) From Proposition 10, we have that at the solution of the MEOp, for any $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$,

$$v^p(s_t) = \frac{\left[s_t^2 + (1-\alpha)(1-2s_t)\right]^2}{2s_t^2 + 4(1-\alpha)(1-2s_t)}, \tag{2.27}$$

which also gives $\frac{\partial v^p(s_t)}{\partial s_t} > 0$. From (2.18), we get that $\frac{\partial s_t}{\partial \tau_t} > 0$. We also know that for $\tau_t > \tilde{\tau}_t$, $s_{t+1} > s_t$ (and equivalently for $\tau_t < \tilde{\tau}_t$, $s_{t+1} < s_t$). Thus, the maximisation of v_1^p leads to $s_2 < s_1$ and given this, the maximisation at $t = 2$ leads to $v_2^p < v_1^p$. Given $\frac{\partial v^p(s_t)}{\partial s_t} > 0$, the maximum value of v_2^p is bounded from the value of s_2, which in turn depends on τ_1.

In this way, from continuity and given $v_2^p < v_1^p$, v_2^p is maximised for $v_2^p = v_1^p = \bar{v}$, with

$$\bar{v} = \frac{\left[s_2^2 + (1-\alpha)(1-2s_2)\right]^2}{2s_2^2 + 4(1-\alpha)(1-2s_2)},$$

where s_2 is given by substituting the dynamic constraint to $v_2^p = v_1^p$ and solving for τ_1.

(ii) Suppose that for all $t \leq T$, $v_1^p = v_2^p = \ldots = v_T^p$. Note that $v_1^p = v_2^p = \ldots = v_T^p$ necessarily implies that $\tau_i \in (\tau_i^*, \tilde{\tau}_i)$ for $i = 1, 2, \ldots, T$; and also that $s_T \leq s_{T-1} \leq \ldots \leq s_1$. Also, from (2.26), it follows that $\frac{\partial \tilde{\tau}_t}{\partial s_t} > 0$.

In order to prove the Lemma, it is sufficient to show that $v_1^p = v_2^p = \ldots = v_{T+1}^p$. Suppose that this is not true. This means that either (a) $v_1^p = v_2^p = \ldots = v_T^p > v_{T+1}^p$ or (b) $v_1^p = v_2^p = \ldots = v_T^p < v_{T+1}^p$.

(a) Let $v_1^p = v_2^p = \ldots = v_T^p > v_{T+1}^p$. Given that $\tau_i \in (\tau_i^*, \tilde{\tau}_i)$ for $i = 1, 2, \ldots, T$, $s_T \leq s_{T-1} \leq \ldots \leq s_1$ and $\frac{\partial \tilde{\tau}_t}{\partial s_t} > 0$, it means that there exists a sequence $\{\hat{\tau}_t\}_1^T$ which gives

$$\hat{v}_1^p = \hat{v}_2^p = \ldots = \hat{v}_T^p < v_1^p = \ldots = v_1^p.$$

Hence, $v_1^p = v_2^p = \ldots = v_T^p > v_{T+1}^p$, which means that this cannot be an IEOp solution.

(b) Let $v_1^p = v_2^p = \ldots = v_T^p < v_{T+1}^p$. Note that at the IEOp solution, $v_1^p = v_2^p = \ldots = v_T^p \geq v^p(s_1, \tilde{\tau}_1)$ and thus $s_T \leq s_1$. Then from (i) we know that at the solution $v_T^p = v_{T+1}^p$.

$\square$

The following result is necessary for the proof of the Theorem which follows.

Lemma 3. *Let* $\alpha \in \left(\frac{1}{2}, 1\right)$, *then for any* $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$ *the following holds:*

$$\frac{\partial v^p(s_t, \tilde{\tau}_t)}{\partial s_t} > 0.$$

Proof. For any $\tau_t \in (0, 1)$,

$$\frac{\partial v^p(s_t, \tau_t)}{\partial s_t} = s_t(1-\tau_t)^2 + \tau_t(1-\tau_t)[2\alpha s_t - 2(1-\alpha)(1-s_t)] > 0,$$

if and only if $s_t(1-\tau_t) + \tau_t[2\alpha s_t - 2(1-\alpha)(1-s_t)] > 0$.

This is equivalent to

$$\tau_t(2 - s_t - 2\alpha) < s_t. \tag{2.28}$$

Here we can distinguish two cases: (i) $2 - s_t - 2\alpha < 0$; and (ii) $2 - s_t - 2\alpha > 0$.

(i) If $2 - s_t - 2\alpha < 0$, then (2.28) holds for any $\tau_t > 0$.

(ii) If $2 - s_t - 2\alpha > 0$, then (2.28) is equivalent to

$$\tau_t < \frac{s_t}{2 - s_t - 2\alpha}.$$

Thus $\frac{\partial v^p(s_t, \tilde{\tau}_t)}{\partial s_t} > 0$ if

$$\tilde{\tau}_t = \frac{s_t - s_t^2}{1 - s_t - \alpha(1 - 2s_t)} < \frac{s}{2 - s - 2\alpha},$$

or

$$(1 - s_t)^2 < \alpha,$$

which is means $s_t > 1 - \sqrt{\alpha}$. In order to conclude the proof is sufficient to prove that

$$s^*(\alpha) = \frac{1}{2}(3 - 2\alpha - \sqrt{4(1-\alpha)^2 + 1}) > 1 - \sqrt{\alpha},$$

or

$$1 - 2\alpha > \sqrt{4(1-\alpha)^2 + 1} - 2\sqrt{\alpha}. \tag{2.29}$$

The left hand side of the above is clearly negative. The right hand side is negative if

$$4(1-\alpha)^2 + 1 < 4\alpha,$$

or equivantly if

$$5 + 4\alpha^2 - 12\alpha < 0,$$

which is true for any $\alpha \in (\frac{1}{2}, 1)$. Thus (2.29) can be expressed as

$$(1-2\alpha)^2 < \left(\sqrt{4(1-\alpha)^2 + 1} - 2\sqrt{\alpha}\right)^2,$$

or after some calculations

$$-4 < -4\sqrt{\alpha + 4\alpha(1-\alpha)^2},$$

which is equivalent to

$$4\alpha(1-\alpha)^2 < 1 - \alpha,$$

or

$$4\alpha(1-\alpha) < 1,$$

which means

$$4\alpha^2 - 4\alpha + 1 > 0,$$

which is true for any $\alpha \in \left(\frac{1}{2}, 1\right)$.

□

This Lemma is quite intuitive as it states that for any level of inequality lower than $s^*(\alpha)$, the level of welfare associated with keeping inequality constant, increases with status.

Theorem 1. *At the IEOp equilibrium:*

1. *For $s_1 \in (0, s^*(\alpha))$, then the sequence $\boldsymbol{\tau}$ is the same as in the solution of the MEOp programme.*

2. *For $s_1 \in \left[s^*(\alpha), \frac{1}{2}\right)$, $s_t = s_1$ for all t.*

Proof. 1. We know from Proposition 10, that for all $s_t \in (0, s^*(\alpha))$, for $\tau_t = \tau_t^*$, $s_{t+1} > s_t$. Thus for any $s_1 \in (0, s^*(\alpha))$, at the solution of (MP1), $v^p(s_1, \tau_1^*)$.

In the same way we can show that given s_1 and the (MP1) solution, at the solution of (MP2) $v^p(s_2, \tau_2^*)$, ..., at the solution of (MPI) for all $I = 2, 3, \ldots$.

2. Suppose not. Then given Lemma 2, this means that there exists a $\hat{\tau}_1 = \hat{\tau}_1(s_1) \in (\tau_t^*(s_1), \tilde{\tau}_t(s_1))$ such that:

$$v^p(s_1, \hat{\tau}_1) > v^p(s_1, \tilde{\tau}_1). \tag{2.30}$$

From the definition of $\tilde{\tau}_t$, the above means that $s_2 < s_1$.

Then from the above and also Proposition 11 and Lemma 3, there exists a $\hat{\tau}_2 = \hat{\tau}_2(s_2) \in (\tau_t^*(s_1), \hat{\tau}_t(s_1))$ such that:

$$v^p(s_1, \hat{\tau}_1) = v^p(s_2, \hat{\tau}_2).$$

Continuing the same process, it is evident that each period, both s_t and $\hat{\tau}_t$ decrease; and tend to $s^*(\alpha)$ and τ_t^*, respectively. Knowing that at $s^*(\alpha)$, $\tau_t^* = \tilde{\tau}_t$, the previous statement means that

$$\lim_{s_t \to s^*(\alpha), \hat{\tau}_t \to \tau_t^*} v^p(s_t, \hat{\tau}_t) = v^p(s^*(\alpha), \tau_t^*) = v^p(s^*(\alpha), \tilde{\tau}_t). \tag{2.31}$$

But, from Lemma 3:

$$v^p(s^*(\alpha), \tilde{\tau}_t) < v^p(s_1, \tilde{\tau}_1), \tag{2.32}$$

for any $s_1 \in \left[s^*(\alpha), \frac{1}{2}\right)$, which contradicts the original hypothesis.

□

This theorem highlights the limitations of the IEOp policy. If initial inequality is lower than a threshold level ($s^*(\alpha)$), then inequality will diminish but will still will be at arguably very high levels, while if inequality is higher than the threshold level, it will not be reduced. The mechanism which drives this result is the relative strength of the incentive channel compared to the redistribution one. This difference in strength between the two channels, lies on the fact that in Greenville, it is allowed for people to have differential education.

In this section we have shown that the EOp ethic in an intergenerational context does not necessarily reduce inequality. If the date of birth of individuals is not seen as a circumstance (myopic programme), very high inequality can be considered as fair if examined through the lens of EOp ethic. If we take into account that the outcomes of one generation define the circumstances of the next, we have shown then even though inequality is not reduced in most cases, it is unfair to grow. These different results highlight the usefulness of the introduction of the IEOp ethical criterion.

2.4 Blueville

2.4.1 Economic Environment

In this city, individuals have same welfare functions as in Greenville

$$u(c, e) = c - \frac{e^2}{2},$$

with

$$c = (1 - \tau)y + \tau\bar{y},$$

where the variables have the same notation as before. There are two differences between the two cities. While in Greenville the productivity of effort, was given by parental status s^i and was different depending on one's social origins, in Blueville everyone has the same productivity s_0, i.e. for $i = p, r$

$$y = s_0 e.$$

For simplicity we will assume that $s_0 = \frac{1}{2}$. The second difference has to do with the fact that in Blueville individuals have false beliefs about the productivity of their effort which depend on their background in the same way as in Greenville. Thus, a person from social class i believes that if they exert effort e_t they will receive (pre-tax) income $s_t^i e_t$, where

$$s_t^i = \frac{c_{t-1}^i}{c_{t-1}^i + c_{t-1}^j}.$$

In this way, in Blueville status s^i affects only the incentives of individuals to put more or less effort but not their actual productivity. This (behavioural) assumption allows us to separate the effects from incentives from the effects from actual productivity and it can be interpreted as the effects of social background to the *confidence* that people regarding their actual abilities.

Optimal Effort

Given that the individuals choose their effort based on their expectation of after tax income, the maximisation problem of individuals, will lead to the same optimal effort as before:

$$e^p = (1-\tau)s$$

and

$$e^r = (1-\tau)(1-s),$$

where $s = s^p$ and $1 - s = s^r$. In this case, the actual pre tax income will be

$$y^r = \frac{(1-\tau)(1-s)}{2} \tag{2.33}$$

for the rich; and

$$y^p = \frac{(1-\tau)s}{2} \tag{2.34}$$

for the poor. The fact that people do have the same productivity of effort but different incentives which are related to their background, raises some issues on whether their differences should be regarded as circumstances or not. On one hand individuals do not have different skills or resources, while on the other they have different behaviour due to their social origins. Thus there could be arguments both in favour of Utilitarian policy and in favour of a Rawlsian EOp one. Below we show that this is not an issue in the current framework.

2.4.2 Single Period Problem

The programme of the EOp Social Planner is the same as in Greenville with the difference that income, consumption and indirect utility of individuals are different due to the effects of the difference in confidence. Consider a one period problem.

Proposition 12. *The EOp equilibrium is the same as the Utilitarian one.*

Proof. Average income is

$$\bar{y} = \frac{1-\tau}{2}[\alpha s + (1-\alpha)(1-s)],$$

or

$$\bar{y} = \frac{1-\tau}{2}(1 + 2\alpha s - \alpha - s). \tag{2.35}$$

Thus consumption is now given by

$$c^p = \frac{(1-\tau)^2 s}{2} + \frac{\tau(1-\tau)}{2}(1 + 2\alpha s - \alpha - s), \tag{2.36}$$

and

$$c^r = \frac{(1-\tau)^2(1-s)}{2} + \frac{\tau(1-\tau)}{2}(1 + 2\alpha s - \alpha - s). \tag{2.37}$$

The indirect utility of the poor (at the optimal effort level), is

$$v^p(\tau) = \frac{(1-\tau)^2(s-s^2)}{2} + \frac{\tau(1-\tau)}{2}(1+2\alpha s-\alpha-s), \tag{2.38}$$

and for the rich is

$$v^r(\tau) = \frac{(1-\tau)^2(s-s^2)}{2} + \frac{\tau(1-\tau)}{2}(1+2\alpha s-\alpha-s). \tag{2.39}$$

Thus $v^p = v^r$. Note that $v^p = v^r$ is concave with respect to τ. At the EOp programme the Social Planner maximizes v^p every period, while in the Utilitarian case the Social Planner maximizes the sum of the utilities of the whole population. For the latter case we observe that

$$\int_i v^i di = \alpha v^p + (1-\alpha)v^r = \alpha v^p + (1-\alpha)v^p = v^p. \tag{2.40}$$

□

This shows that if individuals exert different effort but do not have different educational resources the tax rate that maximises the welfare of the worst off actually maximises the the welfare for the whole of the population. In this case the optimal tax is

$$\tau^* = \frac{1}{2} + \frac{s(1-s)}{2(1+2\alpha s-\alpha-s^2)}. \tag{2.41}$$

Note that the tax rate is always positive.

2.4.3 Myopic Problem

The Myopic EOp programme is defined in the same way as in the previous model.

Proposition 13. *At the solution of the MEOp programme the following are true:*

(i) There exists a unique stationary solution $s^* = \frac{1}{2}$.

(ii) For $s_1 \neq s^*$, at the solution path, s_t converges to s^*.

Proof. We will prove this by showing that for $s_1 \in [0, \frac{1}{2}]$, there exists only one stationary state solution, $s^* = \frac{1}{2}$, which is globally stable.

1. Substitute the optimal tax rate in c_t^p and c_t^r. Then

$$c_t^p = \frac{1}{4}(1-A)\left[(1-A)s + \frac{(1+A)s_t(1-s_t)}{A}\right] \tag{2.42}$$

and

$$c_t^r = \frac{1}{4}(1-A)\left[(1-A)(1-s_t) + \frac{(1+A)s_t(1-s_t)}{A}\right], \tag{2.43}$$

where

$$A = \frac{s_t(1-s_t)}{1+2\alpha s_t - \alpha - s_t^2}. \tag{2.44}$$

2. Calculate $s_{t+1} = \frac{c^p}{c^p+c^r}$. Note that

$$\frac{1}{s_{t+1}} = 1 + \frac{c_t^r}{c_t^p} = 1 + \frac{A(1-A)s + (1+A)s_t(1-s_t)}{A(1-A)(1-s_t) + (1+A)s_t(1-s_t)},$$

then

$$s_{t+1} = \frac{A(1-A)(1-s_t) + (1+A)s_t(1-s_t)}{A(1-A) + 2(1+A)s_t(1-s_t)}. \tag{2.45}$$

3. Notice that for $s = \frac{1}{2}$, $s_{t+1} = s_t$ and that both s_{t+1} and A are not defined for $s_t = 0$.

4. In order to show that $s_t = \frac{1}{2}$ is globally stable, it is sufficient to show that for $s_t \in (0, \frac{1}{2})$, $s_{t+1} - s_t > 0$. Note that this is equivalent to

$$A(1-A)(1-s_t) + (1-A)(1-s_t)s_t > s_t A(1-A) + 2(1+A)s_t^2(1-s_t)$$

or

$$A(1-A)(1-2s_t) + (1-A)(1-s_t)s_t(1-2s_t) > 0,$$

which is true if $A < 1$. Note that $A > 1$, if and only if

$$s_t(1-s_t) < 1 + 2\alpha s_t - \alpha - s_t^2,$$

or

$$1 - \alpha + s_t(2\alpha - 1)$$

which is always true.

□

As we can see from this, contrary to Greenville, in Blueville inequality diminishes and in the long run disappears. But, as we have shown above, the tax rate is also the optimal for the sum of the population (Utilitarian). In this way it does not matter whether date of birth is considered a circumstance or not and thus inequality diminishes not necessarily because it raises the welfare of the worst off but because it raises the welfare of the average individual.

2.4.4 Forward Looking Problem

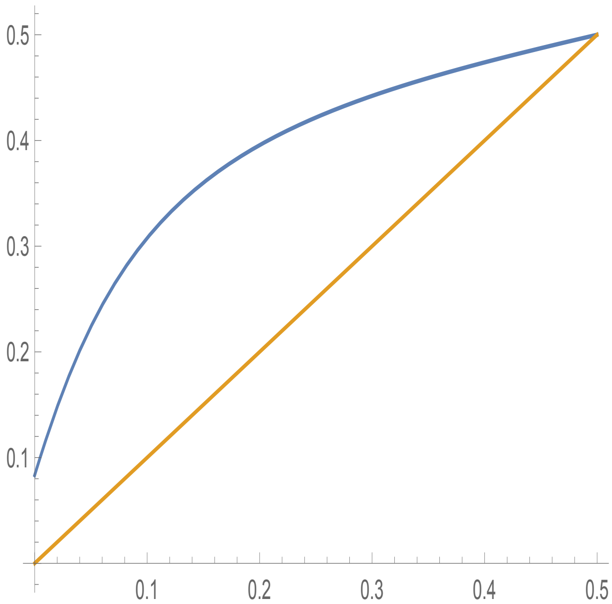
Theorem 2. *For any $s_1 \in (0, \frac{1}{2})$, at the IEOP equilibrium the sequence $\boldsymbol{\tau}$ is the same as in the solution of the MEOp programme.*

Proof. Suppose that this is not true. Let $\boldsymbol{\tau}$, the sequence at the solution of the MEOp. This then means that there exists a $\hat{\boldsymbol{\tau}}$ such that for at least one t, say $t = \mathcal{T}$, there exists $\hat{\tau}_{\mathcal{T}}$, $v^p(\tau^*_{\mathcal{T}}) < v^p(\hat{\tau}_{\mathcal{T}})$ and for $t \neq \mathcal{T}$ $v^p(\tau^*_t) \leq v^p(\hat{\tau}_t)$. From Proposition 13, we know that this cannot hold.

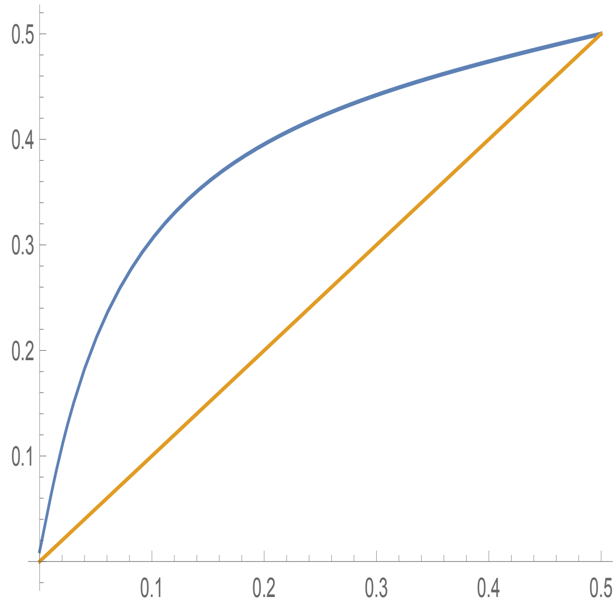
$\square$

The graphs below, present the evolution of inequality for the different cases of populations of rich and poor.

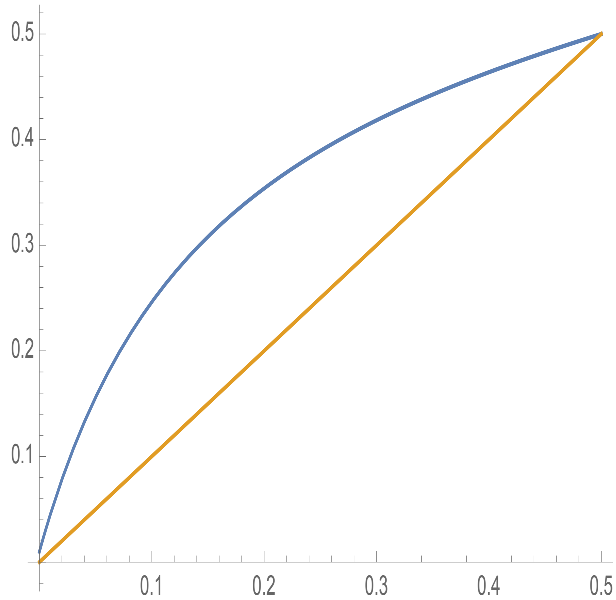
- $\alpha = 0.9$



- $\alpha = 0.99$



- $\alpha = 0.999$



In this model the relative population size matters in a different way than in Greenville. Here, even though in all cases the EOp policy leads to equality in the long run, a smaller population share of the rich slows down the convergence rate towards equality. Our results highlight the fact that EOp ethic can have very different implications regarding inequality, depending on the economic environment which is implemented.

2.5 Conclusions

The aim of this paper has been to provide an answer to whether increasing income inequality can be considered as fair, according to the most mainstream theory of distributive justice, namely *Equality of Opportunity.* Building on the existing literature on EOp, status and inequality, we analysed the implications of intergenerational EOp policies, where the extent of inequality between social classes in a given generation defined the circumstances of the next. In this context the EOp policy should take into account both *inter*- and *intra*-generational inequalities. We have shown that in an economy where inequality of one generation affects the marginal returns of effort of the next one, the IEOp policy cannot decrease inequality. In this way our model which shows that it is fair for inequality not to diminish, provides an example (the first to our knowledge) of Rawlsian "fair" inequality. Contrary to this, in the case where status only has effects on individuals' confidence the IEOp policy leads to decreasing inequality which disappears in the long run.

We have shown that depending on the context the EOp policies are implemented, these can lead to extremely different outcomes. What seems rather surprising is the fact that the economy in which inequality diminishes, the EOp programme is the same as the Utilitarian one. As we have argued in section 2 of this paper, the Utilitarian distribution can be regarded as an extreme case of the EOp one- where the society does not recognise any of the individual characteristics as circumstances. These results highlight the limitations of EOp in terms of being able to reduce inequality as either it is not able to, or when this is able, it is not needed because the Utilitarian distribution would give the same results.

Chapter 3

Heterogeneity and Clustering of Defaults

3.1 Introduction

The hedge fund (HF) industry has experienced an explosive growth in recent years. The total size of the assets managed by HFs in 2015 was estimated at US$2.74 trillion [BarclayHedge, 2016]. Due to the increasing weight of HFs in the financial market, failures of HFs can pose a major threat to the stability of the global financial system. The default of a number of high profile HFs, such as LTCM and HFs owned by Bear Stearns [Haghani, 2014], testifies to this.

At the same time, poor performance of HFs—the prelude to the failure of a HF—is empirically found to be strongly correlated across HFs [Boyson et al., 2010], a phenomenon known as "contagion". Moreover, Boyson et al. [2010] point out that the correlation between HFs' worst returns—falling in the bottom 10% of a HF style's monthly returns—remains high, even after taking into account that HF returns are autocorrelated and the effect of the exposure of HFs to commonly known risk factors. The findings of Boyson et al. [2010] support the theoretical predictions of Brunnermeier and Pedersen [2009], who provide a mechanism revealing how liquidity shocks can lead to downward liquidity spirals and thus to contagion[1]. The mechanism that leads to contagion is closely related to the theory of the "leverage cycle", i.e. the pro-cyclical increase and decrease of leverage, due to the interplay between equity volatility and leverage, put forward by Geanakoplos [1996][2].

[1]Other works which study the the causes of contagion in financial markets include Kyle and Xiong [2001], and Kodres and Pritsker [2002].

[2]In fact the theory of leverage cycle, in contrast to other models that endogenise leverage [Brunnermeier and Pedersen, 2009, Brunnermeier and Sannikov, 2014, Vayanos and Wang, 2012] has the

The combination of the dominant role of HFs in the financial system with the possibility of transmission of the risk, not only to other financial organisations but also to the real economy, has placed the operation of HFs under close scrutiny and has highlighted the significance of regulation of the industry. Regulating the HF industry is not an easy task; Designing the appropriate regulation requires a good understanding of many aspects such as the mechanism which generates defaults at the individual level, the mechanism behind contagion, and finally the parameters which determine the persistency of the effect of a default of an individual HF on the industry. Although Brunnermeier and Pedersen [2009] provide the mechanism behind contagion, they overlooked the persistency of the impact of a default of an individual HF. Our paper aims to fill this gap. In particular, we characterise the conditions under which the correlation between HF's defaults is persistent, i.e. defaults are clustered.

We study an economy with heterogeneous interacting agents (HIA) –HFs in our case – in the tradition of Day and Huang [1990], Brock and LeBaron [1996], Brock and Hommes [1997, 1998], Chiarella and He [2002], Thurner et al. [2012] and Poledna et al. [2014] among others.[3] We find that the feedback between market volatility and margin requirements (downward liquidity spiral), is a necessary yet not a sufficient condition for clustering of defaults to occur, as has been suggested by Boyson et al. [2010]. In this work we show that heterogeneity plays a pivotal role in the emergence of clustered defaults: defaults are clustered only if the degree of heterogeneity is sufficiently high.

We develop a simple dynamic model with a representative mean-reverting noise trader and a finite number of HF managers trading a risky asset. We allow for a setup where heterogeneity regarding the demand of the risky asset may be due to different preferences towards risk, disagreement on the expected price of the asset, or disagreement on the volatility of the market. Evidently, market volatility depends on the HFs' trading strategy, which in turn, depends on HFs' demand. In addition, we allow for the HFs to have access to credit, and we endogenise the probability of default by assuming that a HF would choose to default when its portfolio value falls below a threshold.

In this environment we show that when the degree of heterogeneity is sufficiently high, poorly performing HFs are able to absorb shocks caused by fire sales. As a result, they obtain a larger than usual market share, and improve their performance.

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additional merit of making the endogenous determination of collateral possible.

[3]For a detailed relevant literature review see Hommes [2006], LeBaron [2006] and Chiarella et al. [2009].

In this fashion, a default due to exactly their poor performance is delayed, allowing them to remain in operation until the downturn of the next leverage cycle. This leads to the increase of the probability of poorly and high-performing hedge funds to default in sync at a later time, and thus the probability of collective defaults. Formally, we show that for high degree of heterogeneity the default time-sequence shows infinite memory. Using the definition of Andersen and Bollerslev [1997] clustering is determined by the divergence of the sum (or integral in continuous time) of the autocorrelation function (ACF) of the default time sequence, and therefore, the presence of infinite memory in the underlying stochastic process describing the occurrence of defaults. Furthermore, we establish a quantitative connection between the non-trivial aggregate statistics and the presence of infinite memory in the underlying stochastic process governing the defaults of the HFs. The comparison between the theoretical prediction of the asymptotic behaviour of the autocorrelation function (ACF) of defaults and the numerical findings, reveals that our theoretical predictions are valid even in a market with a finite number of HFs and the clustering of defaults is confirmed.

The structure of the rest of the paper is as follows. Section 3.2 discusses the relevant literature. Section 3.3 presents the economic framework that we use. In section 3.4.1 discusses the numerical findings. In Section 3.4.2, we provide analytical results linking the heavy-tailed aggregate density to the observed statistical character of defaults on a microscopic level, and the power-law decay of the ACF of the default time-series of defaults, identifying that defaults are clustered. Finally, section 3.5 provides a short summary with concluding remarks.

3.2 Relevant Literature

Our paper is related methodologically to the HIA literature; and in terms of content, to the literature which studies the effects of leverage on financial stability.[4] Models with HIA can give rise to emergent properties of systems that are able to replicate the empirical trends seen in asset prices, asset returns and their distributions [Lux, 1995, 1998, Lux and Marchesi, 1999, Iori, 2002, He and Li, 2007, Chiarella et al., 2014]. In Levy [2008], spontaneous crashes are a natural property of a market with heterogeneous investors who are inclined to conform to their peers, under the condition that the strength of the conformity effects is large compared to the degree

[4] The present paper focuses on the role of leverage on a microeconomic level and does not discuss the feedback effects with the Macroeconomy. For the latter see Chiarella and Di Guilmi [2011], Ryoo [2010] and references therein.

of heterogeneity of the investors. In other papers, such as Chiarella [1992], Lux [1995] and Di Guilmi et al. [2014] heterogeneity has to do with the different beliefs and trading rules of the agents (fundamentalists and chartists) which can result to asset price fluctuations and market instability.

The set up of our model is similar to Thurner et al. [2012] and Poledna et al. [2014] which study the effects of leverage in an economy with heterogeneous HFs. Thurner et al. [2012] show that leverage causes fat tails and clustered volatility. Under benign market conditions HFs become more leveraged as this is then more profitable. High levels of leverage are correlated with increased asset price fluctuations that become heavy tailed. The heavy tails are caused by the fact that when a HF reaches its maximum leverage limit then it has to repay part of its loan by selling some of its assets. Poledna et al. [2014] use a very similar framework to test three regulatory policies: (i) imposing limits on the maximum leverage, (ii) similar to the Basle II regulations, and (iii) a hypothetical perfect hedging scheme in which the banks hedge against the leverage-induced risk using options. They find that the effectiveness of the policies depends on the levels of leverage and that even though the perfect hedging scheme reduces volatility in comparison to the Basle II scheme, none of these are able to make the system considerably safer on a systemic level.

Our model extends this framework in two directions. Firstly, in our model the behaviour of HFs is not given by heuristics but it is derived from first principles. In both Thurner et al. [2012] and Poledna et al. [2014], HFs are risk neutral and have different demand of the asset given the same information and the same wealth. The characteristic which makes them heterogeneous, is called “aggression” and aims to capture the different responses of the agents to a mispricing signal. Given the risk neutrality assumption, it is impossible to provide a rigorous explanation for the difference in aggression. Furthermore, deriving the HFs demand functions from first principles: (i) we bridge the gap between Thurner et al. [2012] and Poledna et al. [2014]; and the rest of the leverage cycle literature discussed below and (ii) we provide a framework which allows the study of different types of heterogeneity.

The leverage cycle models start with the collateral equilibrium models of Geanakoplos [1996] and Geanakoplos and Zame [1997] who provide a general equilibrium model of collateral. The key idea behind these models is that lenders require a collateral from the borrowers in order to lend them funds. This borrowing and lending is agreed through a contract of a promise of paying back the loan in future states, where the investor who sells the contract is borrowing money –using a collateral to back the promise– from the agent who buys the contract. Each contract is chosen from a menu of contracts with different loan to value (LTV) ratio. In Geanakoplos [1996] scarcity of collateral leads to only a few contracts being traded, which makes leverage (LTV) endogenous. Finally, the investors default when the the value of the collateral is less than the value of the contract that borrowers and lenders have agreed. Geanakoplos [2003] considers a continuum of risk neutral agents with different priors in a binomial economy with two or three states of the world. He shows how changes in volatility lead to changes in equilibrium leverage which in turn have a bigger effect in asset prices than what agents believe to be the effect of news. Geanakoplos [2003, 1996] show that in some cases all agents will choose the same contract from the contract menu. This result has been recently extended by Fostel and Geanakoplos [2015] who study in more detail the relationship between leverage and default and prove that in all binomial economies with financial assets, exactly one contract is chosen.

Fostel and Geanakoplos [2008] extend the economy of Geanakoplos [2003] to an economy with multiple assets and two risk averse agents instead of a continuum of risk neutral ones; and develop an asset pricing theory which links collateral and liquidity to asset prices. Geanakoplos [2010] combines the insights from Geanakoplos [1996] where the collateral is based on non financial assets and Geanakoplos [2003] where the collateral is based on financial assets; and shows that the introduction of CDS contracts reduces the asset prices. By doing this he puts forward a model of a *double leverage cycle*, in housing and securities, which contributes in the explanation of the 2007-08 crisis. Fostel and Geanakoplos [2012] provide a further analysis of CDS contracts and show: (i) why trenching and leverage initially raised asset prices and (ii) why CDSs lowered them later. Simsek [2013a] considers a continuum of states and two types of agents beliefs, namely optimist and pessimist. He shows that the type of disagreement between agents has more important effects on asset prices than the degree of disagreement between optimists and pessimists.[5]. To our knowledge, this is the only paper in this literature which considers the effect of different degrees of heterogeneity.[6]

Along similar lines the effects of leverage have been studied by Gromb and Vayanos [2002], Acharya and Viswanathan [2011], Brunnermeier and Pedersen [2009], Brunnermeier and Sannikov [2014] and Adrian and Shin [2010], among others. These

[5] Other works in the leverage cycle literature include Geanakoplos and Zame [2014], Geanakoplos [2014] and Fostel and Geanakoplos [2016]. For a recent review of this literature see Fostel and Geanakoplos [2014].

[6] In a different context Simsek [2013b] shows that the level of belief disagreement affects the average consumption risks of individuals in a model which studies the effect of financial innovation on portfolio risks.

approaches differ from the models mentioned in the previous paragraphs in two key aspects. The models of Acharya and Viswanathan [2011], Adrian and Shin [2010], Brunnermeier and Sannikov [2014] and Gromb and Vayanos [2002] focus on the ratio of an agent's total asset value to his total wealth (investor based leverage) while the leverage cycle models of Geanakoplos and coauthors[7] focus on LTV. The second aspect has to do with the fact that in the models of Brunnermeier and Pedersen [2009] and Gromb and Vayanos [2002] the leverage ratio is exogenously given where in the former is given by a *VaR* rule while in the latter it is given by a maximin rule used to prevent defaults. In the cases of Brunnermeier and Sannikov [2014], Acharya and Viswanathan [2011] and Adrian and Shin [2010] leverage is endogenous but is not determined by collateral capacities. In Acharya and Viswanathan [2011] and Adrian and Shin [2010] leverage is determined by asymmetric information between borrowers and lenders; while in Brunnermeier and Sannikov [2014] it is determined by agents' risk aversion.

3.3 Model

3.3.1 Environment

We study an economy with two assets, one riskless (cash C) and one risky, two types of traders and a bank. The supply of the risky asset, which can be viewed as a stock, is fixed and equal to N, whereas there is an infinite supply of the riskless asset. The price of the riskless asset is normalised to 1, whereas the price of the risky asset at time t p_t is determined endogenously. The riskless and the risky asset are traded by a representative, mean-reverting noise trader and K types of hedge funds (HFs), whose objective is to exploit potential mispricings of the risky asset. The role of the bank, which is infinitely liquid, is to provide credit to HFs, by using the HF's assets as collateral.

Representative Hedge Fund: Each HF is run by a myopic portfolio manager, whose objective is to maximise her next period's CRRA utility function over his wealth, W_t:

$$U\left(W_t^j\right) = {W_t^j}^{1-\alpha} / \left(1-\alpha\right), \tag{3.1}$$

where $\alpha > 0$ is the measure of relative risk aversion, and $j \in \{1, \ldots, K\}$.

The manager's strategy of the jth HF is a mapping from her information set S^j to trading orders for the risky and the riskless asset, where D_t^j (C_t^j) denotes the units

[7] Also the models of Brunnermeier and Pedersen [2009], and Simsek [2013a] use the same ratio.

of the risky (riskless) asset the jth HF is willing to trade. Thus, beliefs about the mean logarithmic price of the risky asset $\mathcal{E}[\log(p_{t+1}]$ and the volatility $\mathrm{Var}[\log p_{t+1}]$ plays a crucial role in determining orders.

We assume that only part $(1-\gamma)$ of the current wealth of the HF is available for re-investment in the next period. The purpose of this assumption is to exclude unrealistic cases where the wealth of HFs explodes and default never occurs.[8] This assumption could be interpreted in multiple ways. For instance, it is consistent with the empirical evidence indicating that the compensation of the fund managers is tied to the wealth of the HF. This evidence is also in line with the theoretical literature on optimal contracting in principal-agent environments. Alternatively, the share γ which is not re-invested could be capturing the HF investors' payment. Taking this into account the wealth of a HF evolves according to:

$$W_{t+1}^j = (1-\gamma)W_t^j + (p_{t+1} - p_t)D_t^j, \tag{3.2}$$

where the first term of the RHS captures the value of the portfolio held in the previous period, and the second term captures the change in the value of the risky assets.

It is worth highlighting that the amount of cash required to complement the trading order for a risky asset, i.e., $D_t^j p_t$, may exceed the cash which is available at the beginning of each trading period. This can be the case because we allow for access to credit. However, this access to credit is not unbounded, and is assumed to be subject to regulation. Here the HF cannot become more leveraged than λ_{max}, a maximum ratio of the market value of the risky asset held as collateral by the bank to the net wealth of the risky asset. Thus, the maximum leverage constraint translates into:

$$D_t^j p_t / W_t^j \leq \lambda_{max}.$$

Consequently, the maximum demand for the risky asset is given by:

$$D_{t,\max} = \lambda_{max} W_t^j / p_t, \ \forall j \in \{1, \ldots, K\}. \tag{3.3}$$

Furthermore, we allow the HFs to take only long positions, i.e., to be active only when the asset is underpriced[9].

[8] It is worth highlighting that assuming that the share of wealth which is not re-invested is fixed and constant over time, allows us to develop a more tractable model. However, the critical component for our main findings to go through is that not all wealth is re-invested.

[9] We do this in order to highlight that, even with the HFs taking only long positions, a strategy inherently less risky than short-selling, the clustering of defaults, and thus systemic risk, is still

Default: We define as default any event in which the wealth of a HF falls below $W_{\text{min}} \ll W_0$, where W_0 denotes the initial endowment of each HF upon entrance in the market. This enables us to endogenise the probability of default of each HF. The main objective of this paper is to study both the individual (HF) and collective (systemic) default probabilities over time. After $T_r \sim \text{U}[b, c]$, time-steps the bankrupt HF is replaced by a HF with identical characteristics. This allows us to maintain the character of the market (at a statistical level).

Noise traders: The second type of traders is noise-traders, who are supposed to trade for liquidity reasons. Following the related literature, we assume that the demand d^{nt} of the representative noise-trader for the risky asset, in terms of cash value, is assumed to follow a first-order autoregressive [AR(1)] process [Xiong, 2001, Thurner et al., 2012, Poledna et al., 2014].

$$\log d_t^{\text{nt}} = \rho \log d_{t-1}^{nt} + (1 - \rho) \log(VN) + \chi_t, \tag{3.4}$$

where $\rho \in (0, 1)$ is a parameter controlling the rate of reverting to the mean. Given that the expected value of χ_t and the auto-covariance function are time-independent, the stochastic process is wide-sense stationary, $\chi_t \sim \mathcal{N}\left(0, \sigma_{\text{nt}}^2\right)$, and V is the fundamental value of the risky asset[10].

Trading orders and Equilibrium prices: Finally, the price of the risky asset is determined endogenously by the market clearing condition [together with Eqs. (3.2), 3.4), and (3.7)][11].

$$D_{t+1}^{\text{nt}}(p_{t+1}) + \sum_{j=1}^{K} D_{t+1}^{j}(p_{t+1}) = N, \tag{3.5}$$

where $D_{t+1}^{\text{nt}}(p_{t+1}) = d_{t+1}^{\text{nt}}/p_{t+1}$ stands for the demand of the noise traders whereas $D_{t+1}^{j}(p_{t+1})$ stands for the demand of the jth HF. Both values are in number of shares.

Source of Heterogeneity: A critical component, which lies at the heart of our analysis, is heterogeneity across HFs. We allow for a setup where different HFs respond differently when facing the same price. In particular, we assume that for a given price p_t, different HFs post different demand orders of the risky asset, i.e., $D_t^i \neq$

present if heterogeneity among the prior beliefs is sufficiently large.

[10]The demand of the noise traders in terms of the number of shares of the risky asset D^{nt} and the price of the risky asset p_t at period t is $d_t^{nt} = D_t^{nt} p_t$. Hence, In the absence of the HFs, from Eq. (3.4), and Eq. (3.5) we have $\mathcal{E}\left[\log p_{t+1}\right] = \log V$.

[11]This system of equations is highly non-linear, and thus, can only be solved numerically.

D_t^j for ß $\neq j$. One can think of many cases which could justify heterogeneity across HFs. One explanation could be that HFs have different beliefs about the fundamental value V of the asset. Another case which could justify this heterogeneity could be that HFs agree on the mean, but the disagree on the variance, i.e., $\text{Var}[\log p_{t+1}|\mathcal{F}^j]$. Finally, HFs' heterogeneity might be driven by different degrees of risk aversion, i.e., α. The main findings are qualitatively equivalent independently of which of the previous possible interpretations is implemented. Throughout the paper we assume that HFs disagree on the market volatility.

The rationale behind the assumption that the managers agree on the fundamental value of the asset, but disagree on price volatility, relies on the fact that the fundamental value, as opposed to price volatility, is not affected by the behaviour of HFs. In other words, the fundamental value of the asset is exogenously determined, whereas the volatility of the market is endogenously determined, with its value depending on the HFs' trading strategy, which in turn, depends on their private information set. Hence, it is not feasible for the managers to reach an agreement on the market volatility, because they have access to different information sets, and the market volatility is affected by the information each manager has access to.

Timing: Each period t consists of 4 sub-periods

1. The managers set their demand orders for the risky asset.
2. The price of the risky asset is determined, and the return of each portfolio is realised.
3. The managers receive their compensation.
4. The next-period's wealth is determined.

3.3.2 Optimal Demand

The manager of the jth HF maximises his expected utility, given his beliefs $\mathcal{F}^j$ about the asset's fundamental value and the volatility of the market, and subject to the constraint that the demand cannot exceed $D_{t,max}^j$. This is expressed as

$$D_t^j = argmax_{D_t^j \in [0, D_{t,\max}]} \left\{ \mathcal{E} \left[U(W_{t+1}^j) | \mathcal{F}^j \right] \right\} \tag{3.6}$$

Solving the optimisation problem we obtain[12]

$$D_t^j = \min \left\{ \frac{1}{a} \left(s_j \log (V/p_t) + \frac{1}{2} \right), \lambda_{\max} \right\} \frac{W_t^j}{p_t}, \tag{3.7}$$

[12]For details see Appendix A.

where $s_j = 1/\text{Var}\left[\log p_{t+1}|\mathcal{F}^j\right]$. Therefore, the demand of the HFs is proportional to the expected logarithmic return and their wealth, and inversely proportional to the conditional variance of the logarithm of the price, given their beliefs.

The clustering of HFs' defaults is determined by the decay rate of the of the default time-sequence autocorrelation function (ACF) $C(t')$, with t' being the time-lag variable. If defaults are clustered, then $C(t')$ decays in such a way that the sum of the ACF over the lag variable diverges [Baillie, 1996].

Definition 2. *Let $C(t')$ denote the autocorrelation of the time series of defaults, with t' being the lag variable. Defaults are clustered if and only if*

$$\sum_{t'=0}^{\infty} C(t') \to \infty. \tag{3.8}$$

Given that the ACF is bounded in $[-1, 1]$, it follows that the convergence of the infinite sum is in turn determined by the asymptotic behaviour $t' \gg 1$ of the ACF. In this limit, the sum can be approximated by an integral.

In the following we assume that the ACF of the default time sequence can be approximated by a continuous function for $t' \gg 1$. Then it follows that,

Remark 1. *Defaults are clustered if the ACF asymptotically approaches zero not faster than $C(t') \sim 1/t'$. In this case defaults are interrelated (statistically dependent) for all times.*

Remark 2. *If the decay of the ACF is faster than algebraic, then defaults are not clustered. The effect of the shock caused by the default of a HF on the market is only transient, and the defaults are in the long-run statistically independent.*

Our main goal is to study the relationship between the degree of heterogeneity κ, identified with the difference between extreme values of s_j, and clustering of defaults. The question arises as to whether the leverage cycle is a sufficient condition for the defaults to be clustered, or rather whether there exists a critical value for the degree of heterogeneity above which the mechanism of the leverage cycle leads to clustering of defaults.

In the next section, we present the results of the model. The first subsection presents the numerical results obtained by iterating the model defined above. We present the ACFs for various values of κ and interpret these in light of Remarks 1 and 2. Section 3.4.2 provides an analytical insight into the numerical results.

3.4 Results

Choice of Parameters

In all simulations we consider a market with $K = 10$ HFs. In the following we assume homogeneous preferences towards risk across HFs, and set $a_j = 3.2\ \forall j \in \{1, \dots, 10\}$, this being a typical value for HFs [Gregoriou et al., 2007, p. 417]. From Eq. (3.4) we have $\tilde{\sigma}^2_{\mathrm{nt}} = \sigma^2_{\mathrm{nt}}/(1-\rho^2)$, where ρ is the mean reversion parameter. The inverse of the expected volatility given the HF's prior beliefs, i.e. $s_j = 1/\mathrm{Var}\left[\log p_{t+1}|\mathcal{F}^j\right]$ determines the responsiveness of the HFs to the observed mispricing. In our numerical simulations s_j is sampled from a uniform distribution in $[1, \delta]$, and $\delta \in [1.2, 10]$.

Moreover, the maximum allowed leverage λ_{max} is set to 5. This particular value is representative of the mean leverage across HFs employing different strategies [Ang et al., 2011]. The remaining parameters are chosen as follows: $\sigma^2_{nt} = 0.035$, $V = 1$, $N = 10^9$, $W_0 = 2 \times 10^6$, $W_{\mathrm{min}} = W_0/10$, $\rho = 0.99$ [Poledna et al., 2014], and $\gamma = 5 \times 10^{-4}$. Bankrupt HFs are reintroduced after T_r periods, randomly chosen according to a uniform distribution in $[10, 200]$. Furthermore, the HFs anticipate that their actions—buying when the risky asset is undervalued—will help moderate the fluctuations realised in the market. In other words, all HFs correctly believe that the volatility of the market will be reduced when they enter the market, in comparison to the volatility observed when only the noise traders are active. However, they are uncertain about their collective market power, and therefore the extent to which they will affect the realised volatility. Thus, all HFs believe that $\mathcal{E}\left[\mathrm{Var}\left(\log p_{t+1}\right)\right] < \sigma^2_{nt}/(1-\rho^2)$.

3.4.1 Numerical results

As aforementioned, the leverage cycle consists in the interplay between the variability of prices of the assets put as collateral, and margin requirements. When prices are high, assets used as collateral are overpriced, and creditors are willing to lend. In the face of an abrupt fall of the market price of the assets used as collateral, creditors force the lenders to repay part of the loan, such that the margin requirements are met. Consequently, the lenders are forced to sell in a falling market, accelerating and reinforcing the fall of the price of the collateral, creating thus a vicious cycle.

In our model, a fall in the price of the risky asset used as collateral is caused by a sudden drop of the demand of the noise traders d^{nt}_t. This results into a sudden increase of the leverage ratio of the jth HF, λ^j_t. In case λ^j_t exceeds the margin requirement $\lambda_t^j \leq \lambda_{\max}^j$ HFs are forced to sell, pushing the price even lower. This is illustrated in Fig. 3.1, where we present: (a) the wealth of three HFs (under, moderately, and highly responsive to mispricings, $j = 2, 6, 10$) (b) the corresponding leverage ratio (c) the demand of the noise traders, and (d) the price of the risky asset at equilibrium as a function of time, for a low degree of heterogeneity $\kappa = 0.5$.

At time $t = 738$ [marked by a blue triangle in panel (c)] a drop in the demand of the noise-traders causes an underpricing of the risky asset backing up the loans of HFs [panel (d)]. In turn, the leverage ratio of all the HFs depicted in Fig 3.1(b) $\lambda_{t=738}$ increases abruptly [panel (b)], and the margin requirement $\lambda_{\max} = 5$ becomes binding for the most responsive of the HFs depicted ($j = 6, 10$). At this point, the HFs are forced to deleverage pushing the price of the collateral further down, leading all HFs depicted to default [panel (a)]. The pressure on the price of the risky asset due to the synchronous deleveraging of the highly responsive HFs can clearly be recognized if we compare the lowest price reached around the downturn of the leverage cycle at about $t = 738$ [marked by a the dashed red line in panel (d)], with the equilibrium price at $t = 7153$ [blue filled circle], where the demand of the noise trader becomes virtually the same to that at $t = 738$ [marked by a blue triangle], but the price remains at a considerably higher level. This is because the wealth of all HFs in this case, is such that the leverage ratio stays well below the maximum threshold [see panel (b)], and the leverage cycle mechanism remains inactive.

Another observation worth commenting on, is the fact that after the HFs have been reintroduced in the market, we notice that the least responsive HF ($j = 2$), defaults another 2 times, by the end of the time-series depicted in Fig. 3.1, namely at $t = 3976$, and $t = 9161$ [also marked by blue triangles in panel (a)]. not because of the presence of a shock in the demand of the risky asset, but rather, due to its poor performance. This is because time is costly in our model (HFs pay managerial fees), and if the profitability of a HF is low, then it will inevitably be led to bankruptcy, even in the absence of a shock on the demand of the risky asset. These defaults happen at random times, i.e. when the observed mispricings happen to be small, or when the asset is overpriced, for a period of time, and the profits made are also small, or null, respectively. This also explains the second default of the 6th HF, at $t = 6618$ [red triangle in Fig. 3.1(a)], when all the HFs are well below the maximum leverage constraint.

Let us now study an example with a higher degree of heterogeneity. In Fig. 3.2 we present the wealth W_t^j [panel (a)], the leverage ratio λ_t^j [panel (b)] of 3 representative HFs [$j = 2, 6, 10$], as well as the logarithmic returns [panel (c)] as a function of time,

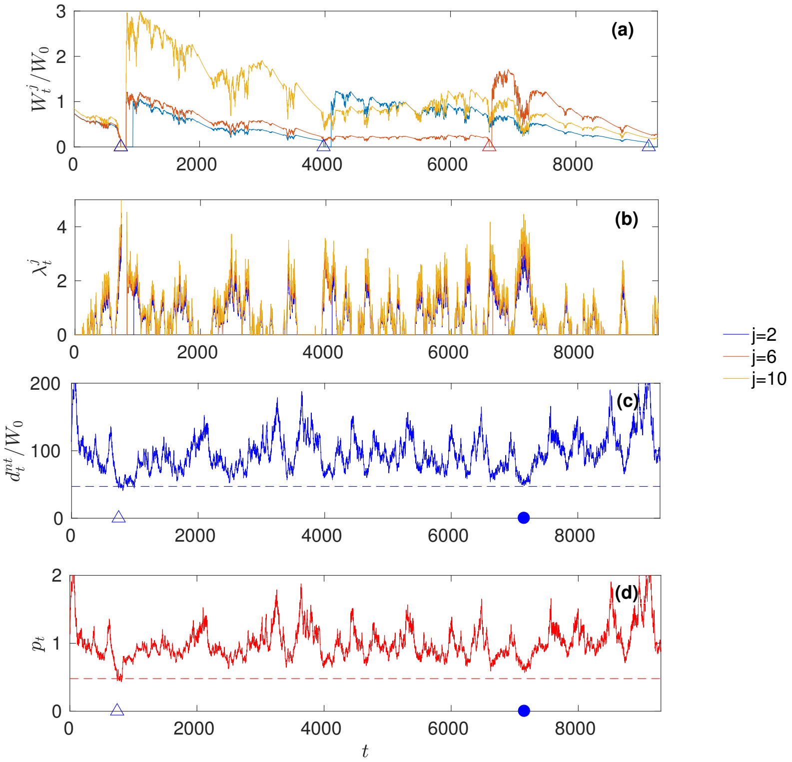

Figure 3.1: (a) The wealth normalised by the endowment W_0, (b) the leverage ratio λ_t^j, (c) the demand of the noise traders in terms of money-value, normalised also by W_0, and (d) the equilibrium price of the risky asset, as a function of time, in the case of $\kappa = 0.5$.

for $\kappa = 3$. At $t = 493$ [marked by a red circle in panels (a), and (c)] the leverage cycle becomes active, causing an underpricing of the risky asset. However, the least responsive to mispricings hedge fund ($j = 2$) of the three depicted, manages to absorb the shock, as it stays below the maximum leverage $\lambda_{\max} = 5$ [see panel (b), blue line], and never receives a margin call. However, the bankruptcy of the more responsive HFs, offers the HF that has survived the shock ($j = 2$), the opportunity to seize a larger market share and, as a result, to perform better in the short-run, restoring its wealth to a level similar to the one before the shock occurred. In this way, the most poorly performing HF is given the opportunity to continue operating until the next downturn of the leverage cycle, at which point it defaults along with the rest of the HFs at $t = 2371$ [red disc]. After the second crash of the market we observe the end of yet another leverage cycle, at which point all the depicted HFs default again in sync at $t = 3044$ [black disc]. The narrative is repeated once more at $t = 3684$ [blue circle], when again the least responsive HF after absorbing the shock gets a larger market share, increasing shortly its profitability.

In conclusion, the study of time-series in the case of low ($\kappa = 0.5$) and high heterogeneity ($\kappa = 3$) reveals that increased heterogeneity leads to the increase of collective defaults. Even more, the synchronous default of highly responsive HFs, gives the opportunity to the less responsive ones to increase their market share, and thus, their profitability, even for a short-period of time. Still, this increases the chance of the poor-performing HFs to survive until the next downturn of the leverage cycle, suppressing defaults occurring at random times due to their poor performance, and thus increasing even more the probability of synchronous defaults. Therefore, this analysis hints that the degree of heterogeneity is intimately connected to the level of systemic risk in the market.

To assess quantitatively the effect of the degree of heterogeneity, explained above, on the systemic risk, we study the persistence of the correlation between defaults [see definition 2]. In Figure 3.3(a) we compare the numerically computed ACF of the default time-sequence[13] as observed on the aggregate level for 11 different degrees of heterogeneity κ, determined by the support of the distribution of s_j. The results were obtained by iterating the model described in Section 3.3 for up to 3×10^8 periods, and averaging over 40 realisations of the responsiveness s_j; namely, $s_j \sim \mathrm{U}[1, \delta]$, with $\delta = \{1.2, 1.4, 1.7, 2, 3, 5, 6, \ldots, 10\}$. Clearly, when the degree of heterogeneity $\kappa = \delta - 1 \leq 1$, the ACF decays far more rapidly in comparison with larger values of heterogeneity. In fact, as it can be observed in the figure, the ACF

[13] The time-sequence considered is constructed by mapping defaults to 1s, irrespective of which HF defaulted, and to 0 otherwise.

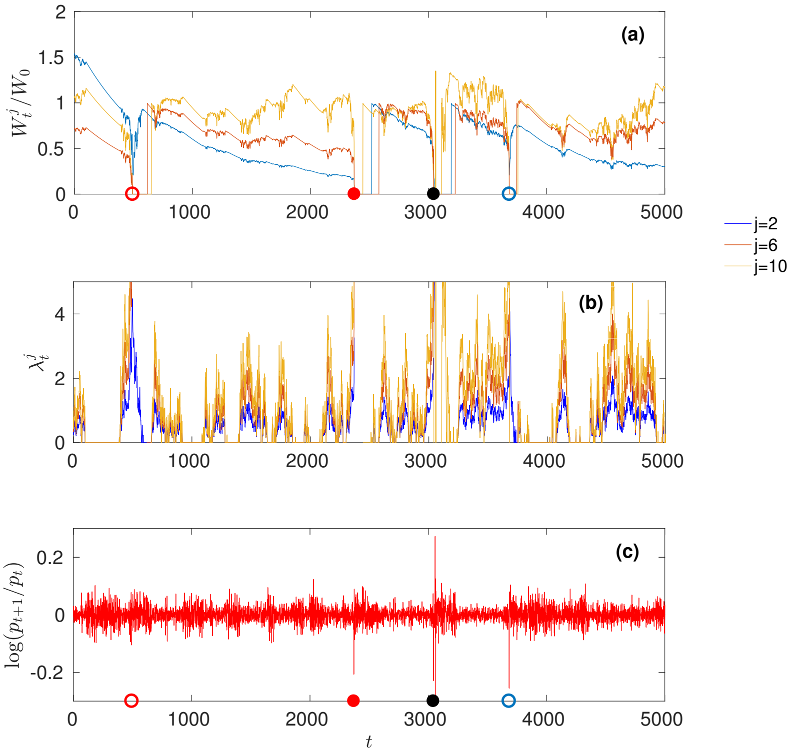

Figure 3.2: (a) The wealth normalised by the endowment W_0, (b) the leverage ratio λ_t^j, and (d) the logarithmic returns on the risky asset, as a function of time, with $\kappa = 3$.

for $\kappa \leq 1$ decays faster than a power-law with exponent equal to -1 (black dashed line), which is the largest exponent (in absolute terms) leading to a non-integrable ACF [see Remark 1]. On the other hand, the converse is true for large degrees of heterogeneity ($\kappa > 2$), in which case the ACF decays asymptotically—$t' \gg 1$—as a power-law with exponent less than 1 in absolute value. Consequently,

Result 1. *For $\kappa \leq 1$, the mechanism of the leverage cycle, does not result into sufficiently high long-range correlations for defaults to be clustered.*

Figure 3.3(a) also shows that for increasing heterogeneity the ACF converges to a limiting form as the heterogeneity is increased, which is reflected in the coalescence of the ACFs corresponding to $\kappa \geq 5$. The latter is more clearly demonstrated in Fig 3.3(b), where a blow-up of the area within the rectangle shown in panel (a) is presented. Therefore,

Result 2. *For sufficiently large values of the degree of heterogeneity κ, namely for $\kappa \geq 5$, the ACF converges to a limiting form exhibiting a power-law trend with an exponent less than 1 (in absolute value).*

To gain some insight into the qualitative difference with respect to the persistence of correlations between defaults as a function of the degree of heterogeneity κ, let us turn our attention to the default statistics. In Fig. 3.5 we present the aggregate PDF of waiting times between defaults[14] using a logarithmic scale on both axes for 6 different values of κ. We observe that for small degrees of heterogeneity $\kappa = \{0.2, 0.4, 0.7\}$ the density function asymptotically decays approximately exponentially. This is better demonstrated in the inset were we use semi-logarithmic axes[15]. On the contrary, for sufficiently large heterogeneity—such that the corresponding ACFs have converged to the limiting form—the PDFs exhibit a constant decay rate in the doubly logarithmic plot (power-law tail). Fitting the aggregate density for $\kappa = 9$[16], corresponding to the highest degree of heterogeneity considered, with the model $\tilde{P}(\tau) \sim \tau^{-\zeta}$ we obtain $\zeta = 2.84 \pm 0.03$ (red dashed line).

Let us now turn our attention to the statistical properties of HFs on a microscopic scale, i.e. study each HF default statistics individually. In Fig. 3.6 we show as an example the density function $P_j(\tau)$, of waiting times τ between defaults, for a number of HFs corresponding to high heterogeneity, $\kappa = 9$, with $s_j = \{2, 4, 6, 8, 10\}$

[14] The PDF of waiting-times between default is also known as the failure function in survival analysis theory.

[15] The use of a logarithmic scale for the vertical axis transforms an exponential function to a linear one.

[16] To increase the accuracy of the fit, we increase the number of realisations of s_j to 10^3.

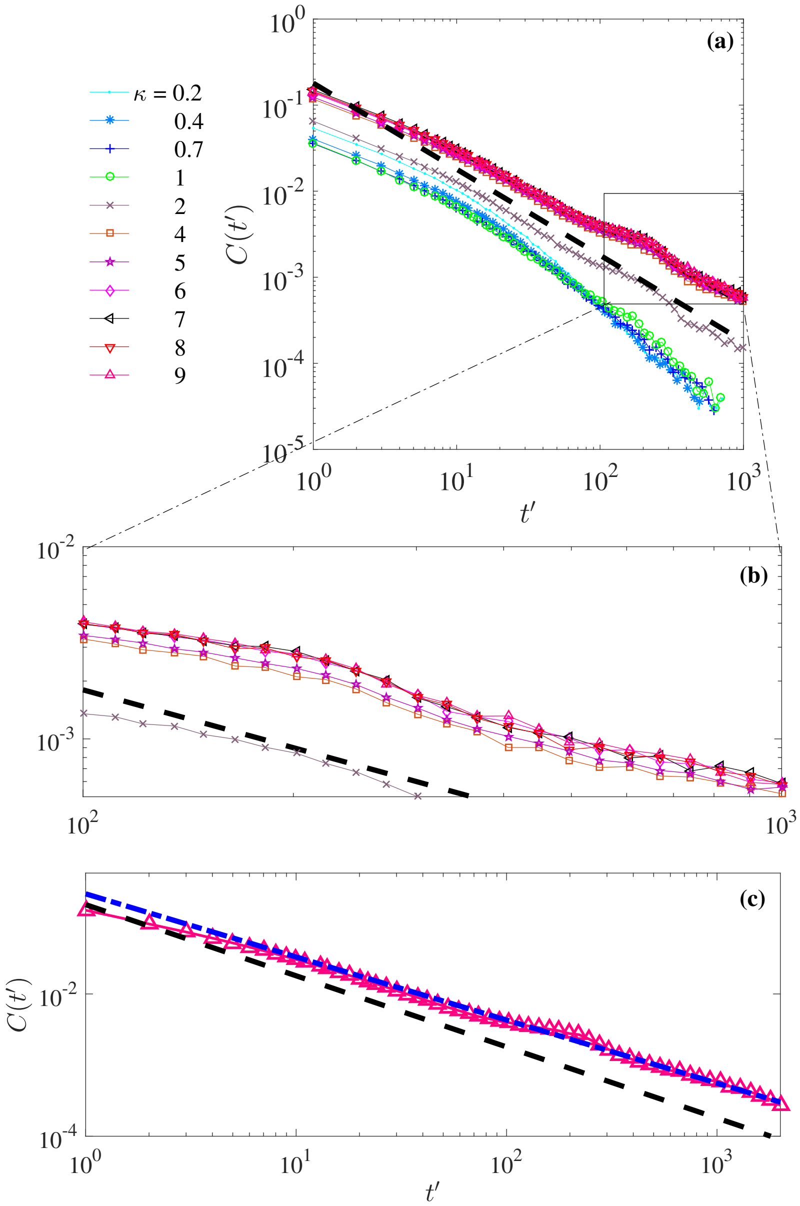

Figure 3.3: (a) The ACF of the binary sequence of defaults corresponding to 11 different values of κ. The dashed black line corresponds to a power-law with exponent -1, which is the largest exponent that leads to clustering [see Remark 1].

Figure 3.4: (b) A blow-up of the rectangular area shown in panel (a) illustrating the coalescence of $C(t')$ for large values of the degree of heterogeneity, $\kappa = \{6, 7, 8, 9\}$. (c) The ACF corresponding to $\kappa = 9$, averaged over 5×10^2 different realisations of s_j (red upright triangles). The blue dot-dashed line is the result of fitting $C(t')$ with a power-law model $C(t') \propto t'^{-\eta}$, $\eta = 0.887 \pm 0.003$ ($R^2 = 0.9927$). The power-law with exponent -1 is also shown for the sake of comparison (black dashed line).

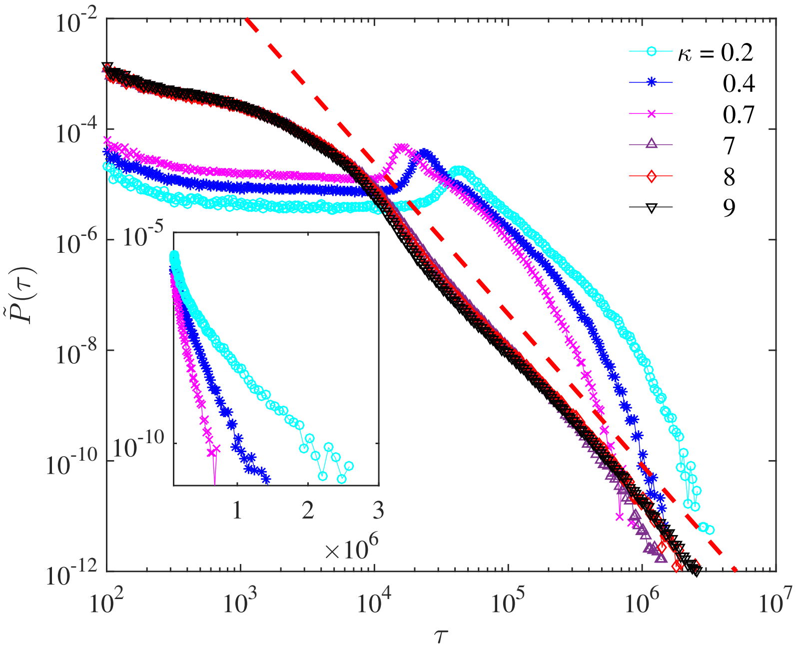

Figure 3.5: The aggregate PDF of waiting times between defaults for 6 different degrees of heterogeneity using double logarithmic scale. For large heterogeneity $\kappa = \{7, 8, 9\}$, we observe that the PDF is decaying approximately linearly, corresponding to a power-law decay. Performing a fit with the model $\tilde{P}(\tau) \sim \tau^{-\zeta}$ we obtain $\zeta = 2.84 \pm 0.03$ ($R^2 = 0.9947$). To illustrate the approximate exponential asymptotic decay of the aggregate PDF for $\kappa = \{0.2, 0.4, 0.7\}$ we also show the corresponding aggregate densities using a logarithmic scale on the vertical axis (inset).

on a log-linear scale. The results were obtained by iterating the model for 3×10^8 periods and averaging over 100 different initial conditions[17], holding s_j fixed at $\{1, 2, \ldots 10\}$. We observe that $P_j(\tau)$ for $\tau \gg 1$ decays linearly, and thus it can be well described by an exponential function.

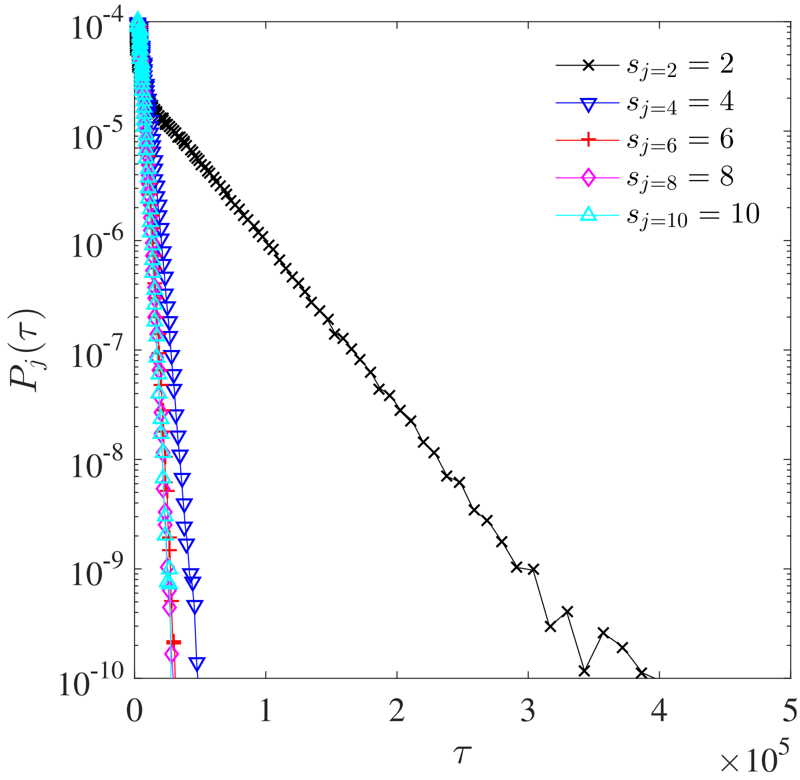

Figure 3.6: The PDF of waiting times between defaults τ for specific HFs, having different responsiveness $s_j = \{2, 4, 6, 8, 10\}$ (black diagonal crosses, downright triangles, red upright crosses, magenta diamonds and cyan upright triangles, respectively). Note the log-linear scale.

Consequently, all HFs on a microscopic level—individually—are characterised by exponential PDFs of waiting-times, and therefore the default events approximately follow a Poisson process. The stability of each HF, quantified by the probability of default per time-step μ_j, is different for each HF, and depends on its responsiveness s_j. This is reflected by the different slopes of the approximately straight lines shown in Fig 3.6 for the different values of s_j.

Thus, the default statistics on an aggregate level are qualitatively different for large values of κ compared to the corresponding ones observed when each HF is studied

[17]We are averaging using different seeds for the pseudo-random number generator used in Eq. (3.4).

individually. Moreover we have already established that for such high values of the degree of heterogeneity the defaults are clustered. In the following we will investigate how the emergence of a fat-tail in the aggregate statistics is connected with the observed clustering of defaults.

3.4.2 Analytical Results

From the numerical results, we observe that $P_j(\tau)$, for $\tau \gg 1$ decays linearly (in log-linear scale) and thus it can be well described by an exponential function. Therefore we can assume that:

$$P_j(\tau; \tau \gg 1) \sim \mu_j \exp(-\mu_j \tau), \ \forall j \ \in \{1, \ldots, 10\} \tag{3.9}$$

When the above is true, we know that for sufficiently long waiting times between defaults; default events of individual HFs have the following statistical properties: (i) they are approximately independent and (ii) occur with a well defined mean probability per unit time step. From this we get that the probability $P_j(T = \tau)$, $\tau \in \mathcal{N}_+$, is given by a geometric probability mass function (PMF)

$$P_j(\tau) = p_j(1 - p_j)^{\tau - 1}, \tag{3.10}$$

where p_j denotes the probability of default of the jth HF.

Given that our focus is in the asymptotic properties of the PDFs, T can be treated as a continuous variable. In this limit, the renewal process given by equation (3.10), becomes a Poisson process; and the geometric PMF tends to an exponential PDF[18]. Thus equation (3.9) can be approximated by (3.10).

Consequently, our intuition with respect to the statistical properties of default events when each HF is considered individually is aligned with our numerical findings presented in the previous Section [see Fig 3.6].

The question then arises as to how the aggregation of these very simple stochastic processes can lead to the non-trivial fat-tailed statistics we observed in Fig 3.5 for a sufficiently high degree of heterogeneity. Evidently, the aggregate PDF $\tilde{P}(\tau)$ we seek to obtain is a result of the mixing of the Poisson processes governing each of the HFs. In the limit of a continuum of HFs the aggregate distribution is

$$\tilde{P}(\tau) = \int_0^\infty \mu \exp(-\mu\tau)\rho(\mu)d\mu, \tag{3.11}$$

[18] This limit is valid for $\tau \gg 1$ and $p_j \ll 1$ such that $\tau p_j = \mu_j$, where μ_j is the parameter of the exponential PDF [see equation (3.9)] [Nelson, 1995].

where $\rho(\mu)$ stands for the PDF of μ given the responsiveness s_j[19].

Assumption 1. *$\rho(\mu)$ in a neighbourhood of 0 can be expanded in a power series of the form $\rho(\mu) = \mu^\nu \sum_{k=0}^{n} c_k \mu^k + R_{n+1}(\mu)$, with $\nu > -1$* [20].

This assumption is quite general, and only excludes functions that behave pathologically in a neighbourhood around 0. Then from equation (3.9) and Assumption 1 we can show that the aggregation of the exponential densities determining the default statistic for each HF individually leads to a qualitatively different heavy-tailed PDF.

Let $\mu^j \in \mathcal{R}^+$ be the mean default rate of the jth HF, contributing at the aggregate level with a statistical weight $\rho(\mu)$, which is determined by the interactions between the agents in the market and the distribution of the responsiveness s.

Proposition 14. *Consider the exponential density function $P(\tau;\mu)$ describing the individual default statistics of a HF. It follows then from Assumption 1, that the aggregate PDF $\tilde{P}(\tau)$ exhibits a power-law tail.*

Proof. The aggregate density can be viewed as the Laplace transform $\mathcal{L}[.]$ of the function $\phi(\mu) \equiv \mu\rho(\mu)$, with respect to μ. Hence,

$$\tilde{P}(T = \tau) \equiv \mathcal{L}[\phi(\mu)](\tau) = \int_0^\infty \phi(\mu) \exp(-\mu\tau) d\mu. \tag{3.12}$$

To complete the proof we apply Watson's Lemma [Debnath and Bhatta, 2007, p. 171] to the function $\phi(\mu)$, according to which the asymptotic expansion of the Laplace transform of a function $f(\mu)$ that admits a power-series expansion in a neighbourhood of 0 [see Assumption 1] of the form $f(\mu) = \mu^\nu \sum_{k=0}^{n} b_k \mu^k + R_{n+1}(\mu)$, with $\nu > -1$ is

$$\mathcal{L}_\mu [f(\mu)](\tau) \sim \sum_{k=0}^{n} b_k \frac{\Gamma(\nu + k + 1)}{\tau^{\nu+k+1}} + O\left(\frac{1}{\tau^{\nu+n+2}}\right). \tag{3.13}$$

Given that $\phi(\mu)$ for $\mu \to 0_+$ is

$$\phi(\mu) = \mu^{\nu+1} \sum_{k=0}^{n} c_k \mu^k + R_{n+1}(\mu), \tag{3.14}$$

we conclude that

$$\tilde{P}(\tau) \propto \frac{1}{\tau^{k+\nu+2}} + O\left(\frac{1}{\tau^{k+\nu+3}}\right). \tag{3.15}$$

[19]The distribution function of the random parameter μ is also known as the *structure* or *mixing* distribution [Beichelt, 2010].

[20]Since $\rho(\mu)$ is a PDF it must be normalisable and thus, a singularity at $\mu = 0$ must be integrable.

□

Corollary 3. *If $0 < k + \nu \leq 1$, then the variance of the aggregate density diverges (shows a fat tail). However, the expected value of τ remains finite.*

An important aspect of the emergent heavy-tailed statistics stemming from the heterogeneous behaviour of the HFs, is the absence of a characteristic time-scale for the occurrence of defaults (scale-free asymptotic behaviour[21]). Thus, even if each HF defaults according to a Poisson process with intensity $\mu(s)$—which has the intrinsic characteristic time-scale $1/\mu(s)$—after aggregation this property is lost due to the mixing of all the individual time-scales. Therefore, on a macroscopic level, there is no characteristic time-scale, and all time-scales, short and long, become relevant.

This characteristic becomes even more prominent if the density function $\rho(\mu)$ is such that the resulting aggregate density becomes fat-tailed, i.e. the variance of the aggregate distribution diverges. In this case extreme values of waiting times between defaults will be occasionally observed, deviating far from the mean. This will leave a particular "geometrical" imprint on the sequence of default times. Defaults occurring close together in time (short waiting times τ) will be clearly separated due to the non-negligible probability assigned to long waiting times. Consequently, defaults, macroscopically, will have a "bursty" or intermittent, character, with long quiescent periods of time without the occurrence of defaults and "violent" periods during which many defaults are observed close together in time. Hence, infinite variance of the aggregate density will result in the clustering of defaults.

In order to show this analytically, we construct a binary sequence by mapping time-steps when no default events occur to 0 and 1 otherwise. As we show below, if the variance of the aggregate distribution is infinite, then the autocorrelation function of the binary sequence generated in this manner, exhibits a power-law asymptotic behaviour with an exponent $\beta < 1$. Therefore, the ACF is non-summable and consequently, according to Definition 2 defaults are clustered.

Let T_i, $i \in \mathcal{N}_+$, be a sequence of times when one or more HFs default and assume that the PDF of waiting times between defaults $\tilde{P}(\tau)$, for $\tau \to \infty$, behaves (to leading order) as $\tilde{P}(\tau) \propto \tau^{-a}$. Consider now the renewal process $S_m = \sum_{i=0}^{m} T_i$. Let

[21] If a function $f(x)$ is a power-law, i.e. $f(x) = cx^a$, then a rescaling of the independent variable of the form $x \to bx$ leaves the functional form invariant ($f(x)$ remains a power-law). In fact, a power-law functional form is a necessary and sufficient condition for scale invariance [Farmer and Geanakoplos, 2008]. This scale-free behaviour of power-laws is intimately linked with concepts such as self-similarity and fractals [Mandelbrot, 1983].

$Y(t) = 1_{[0,t]}(S_m)$, where $1_A : \mathcal{R} \to \{0,1\}$ denotes the indicator function, satisfying

$$1_A = \left\{ \begin{array}{ccc} 1 & : & x \in A \\ 0 & : & x \notin A \end{array} \right\}.$$

Theorem 3. *If the variance of the density function $\tilde{P}(\tau)$ diverges, i.e. $2 < a \leq 3$, then the ACF of $Y(t)$,*

$$C(t') = \frac{\mathcal{E}\left[Y_{t_0} Y_{t_0+t'}\right] - \mathcal{E}\left[Y_{t_0}\right] \mathcal{E}\left[Y_{t_0+t'}\right]}{\sigma_Y^2},$$

where t_0, $t' \in \mathcal{R}$ and σ_Y^2 is the variance of $Y(t)$, for $t \to \infty$ decays as

$$C(t') \propto t'^{2-\alpha} \tag{3.16}$$

Proof. Assuming that the process defined by $Y(t)$ is ergodic we can express the autocorrelation as,

$$C(t') \propto \lim_{K \to \infty} \frac{1}{K} \sum_{t=0}^{K} Y_t Y_{t+t'}. \tag{3.17}$$

Obviously, in equation (3.17) for $Y_t Y_{t+t'}$ to be non-zero, a default must have occurred at both time t and t'[22]. The PDF $\tilde{P}(\tau)$ can be viewed as the conditional probability of observing a default at period t given that a default has occurred $t - \tau$ periods earlier. If we further define $C(0) = 1$ and $\tilde{P}(0) = 0$, the correlation function can then be expressed in terms of the aggregate density as follows:

$$C(t') = \sum_{\tau=0}^{t'} C(t' - \tau)\tilde{P}(\tau) + \delta_{t',0}, \tag{3.18}$$

where $\delta_{t',0}$ is the Kronecker delta. Since we are interested in the long time limit of the ACF we can treat time as a continuous variable and solve equation (3.18) by applying the Laplace transform $\mathcal{L}\{f(\tau)\}(s) = \int_0^\infty f(\tau)\exp(-s\tau)d\tau$, utilising also the convolution theorem. Taking these steps we obtain

$$C(s) = \frac{1}{1 - \tilde{P}(s)}, \tag{3.19}$$

where $\tilde{P}(s) = \int_1^\infty \tilde{P}(\tau)\exp(-s\tau)d\tau$, since $\tilde{P}(0) = 0$. After the substitution of the Laplace transform of the aggregate density in equation (3.19), one can easily derive the correlation function in the Fourier space $\mathcal{F}\{C(t')\}$ by the use of the identity

[22] A detailed exposition of the proof is given in Appendix 3.7.

[Jeffrey and Zwillinger, 2007, p. 1129],

$$\mathcal{F}\{C(t')\} \propto C(s \to 2\pi i f) + C(s \to -2\pi i f). \tag{3.20}$$

to obtain ,

$$\mathcal{F}\{C(t')\} \overset{f \ll 1}{\propto} \begin{cases} f^{a-3}, & 2 < a < 3 \\ |\log(f)|, & a = 3 \\ \text{const.}, & a > 3 \end{cases} . \tag{3.21}$$

Therefore, for $a > 3$ this power spectral density function is a constant and Y_t behaves as white noise. Consequently, if the variance of $\tilde{P}(\tau)$ is finite, then Y_t is uncorrelated for large values of t'.

Finally, inverting the Fourier transform when $2 < a \leq 3$ we find that the autocorrelation function asymptotically ($t' \gg 1$) behaves as

$$C(t') \propto t'^{2-a},\ 2 < a \leq 3. \tag{3.22}$$

□

Turning back to the numerical results shown in Fig. 3.5, the aggregate PDF as already discussed converges to a limiting form, characterised by a fat-tail with an exponent equal -2.84 ± 0.03. Therefore, from equation (3.22) we deduce that the ACF should show a power-law trend with exponent -0.84 ± 0.03. The result of the regression of the ACF for $\kappa = 9$ was -0.887 ± 0.003 [blue dashed-dotted line in Fig. 3.3(c)], in good agreement with the analytical result.

In this Section we have shown that when the default statistics of HFs are individually described by (different) Poisson processes (due to the heterogeneity in the prior beliefs among the HFs) we obtain a qualitatively different result after aggregation: the aggregate PDF of the waiting-times between defaults exhibits a power-law tail for long waiting-times. As shown in Proposition 14, if the relative proportion of very stable HFs approaches 0 sufficiently slowly (at most linearly with respect to the individual default rate μ, as $\mu \to 0$), then long waiting-times between defaults become probable, and as a result, defaults which occur closely in time will be separated by long quiescent time periods and defaults will form clusters. The latter is quantified by the non-integrability of the ACF of the sequence of default times, signifying infinite memory of the underlying stochastic process describing defaults on the aggregate level. It is worth commenting on the fact that the most stable (in terms of defaults) HFs are responsible for the appearance of a fat-tail in the aggregate PDF.

3.5 Conclusions

This paper studied the role of the heterogeneity in available information among different HFs in the emergence of clustering of defaults. The economic mechanism leading to the clustering of defaults is related to the leverage cycle put forward by Geanakoplos and coauthors. In these models the presence of leverage in a market leads to the overpricing of the collateral used to back-up loans during a boom, whereas, during a recession, collateral becomes depreciated due to a synchronous deleveraging compelled by the creditors. In the present work we have shown that this feedback effect between market volatility and margin requirements is a necessary, yet not a sufficient condition for the clustering of defaults and, in this sense, the emergence of systemic risk.

We have shown that a large difference in the expectations of the HFs is an essential ingredient for defaults to be clustered. We show that when the degree of heterogeneity, realised in our model beliefs across HFs about the volatility of the market, is sufficiently high, poorly performing HFs are able to absorb shocks caused by fire sales. As a result, they obtain a larger than usual market share, and improve their performance. In this fashion, a default due to their poor performance is delayed, allowing them to remain in operation until the downturn of the next leverage cycle. This leads to the increase of the probability of poorly and high-performing hedge funds to default in sync at a later time, and thus the probability of collective defaults.

This manifests itself in the emergence of scale-free (heavy tailed) statistics on the aggregate level. We show, that this scale-free character of the aggregate survival statistics, when combined with large fluctuations of the observed waiting-times between defaults, i.e. infinite variance of the corresponding aggregate PDF, leads to the presence of infinite memory in the default time sequence. Consequently, the probability of observing a default of a HF in the future is much higher if one (or more) is observed in the recent past, and as such, defaults are clustered. Therefore, our work shows that individual stability can lead to market-wide risk.

Interestingly, a slow-decaying PDF of waiting-times, which inherently signifies a non-negligible measure of extremely stable HFs, is shown to be directly connected with the presence of infinite memory. The leverage cycle theory correctly emphasises the importance of collateral, in contrast to the conventional view, according to which the interest rate completely determines the demand and supply of credit. However, the feedback loop created by the volatility of asset prices and margin constraints poses a systemic risk only if the market is sufficiently heterogeneous such that "pessimistic" players, who individually are very stable, exceed a critical mass.

This work raises several interesting questions, which we aim to address in the future. In this paper we have assumed that the difference in beliefs is due to disagreement about the long-run volatility of the risky asset, and remains constant over time, i.e. the agents do not update their beliefs given their observations. This assumption is crucial in order to be able to analyse the effects of different degrees of heterogeneity. Regarding this issue, future work can take two different directions: On the one hand, this assumption can be relaxed, allowing agents to update their beliefs on market volatility. However, given that market volatility is endogenous, it is not guaranteed that agents' beliefs will convergence. On the other hand, we can study the effects of heterogeneity stemming from different aversion to risk among the HFs, while retaining the common prior assumption. Furthermore, these two approaches can be combined by assuming both different aversion to risk, and different beliefs about price volatility. Finally, our work can also be extended in two further directions. The first being to give a more active role to the bank which provides loans, while the second is to study the effects of different regulations on credit supply.

3.6 Appendix A

We seek to determine the optimal demand for each of the HFs given their beliefs about price volatility $\mathcal{F}^j$. This translates into the optimisation problem, assuming log-normal returns on the risky asset

$$argmax_{D_t^j \in [0, D_{t,\max}]} \left\{ \mathcal{E}\left[U(W_{t+1}^j) | q_j \right] \right\}, \tag{A.1}$$

where $U\left(W_{t+1}^j\right) = {W_{t+1}^j}^{1-a}/(1-a) \sim {W_{t+1}^j}^{1-a}$, and W_{t+1}^j is the wealth of the jth HF at the next period. To simplify the notation, in the following we will assume that the expected value, and variance are always conditioned on HF's prior beliefs, and moreover, we will drop the superscript j. Eq. (A.1) is equivalent to the maximisation of the logarithm of the expected utility. Furthermore, given that returns are log-normally distributed, it follows that [Campbell and Viceira, 2002, pp. 17-21]

$$\log \mathcal{E}\left[{W_{t+1}}^{1-a} \right] = \mathcal{E}\left[\log {W_{t+1}}^{1-a} \right] + \frac{\mathrm{Var}\left[\log {W_{t+1}}^{1-a} \right]}{2} \tag{A.2}$$

Consequently, the problem becomes

$$argmax_{D_t \in [0, D_{t,\max}]} \left\{ (1-a)\,\mathcal{E}\left[\log W_{t+1}\right] + (1-a)^2 \frac{\mathrm{Var}\left[\log W_{t+1}\right]}{2} \right\}. \tag{A.3}$$

The wealth of the jth HF at the next period is

$$W_{t+1} = (1-\gamma)(1 + x_t R_{t+1}) W_t, \tag{A.4}$$

where x is the fraction of its wealth invested into the risky asset, and R the (arithmetic) return of the portfolio. Re-expressing Eq. (A.4) in terms of the logarithmic returns r we get

$$\log\left(W_{t+1}\right) = \log W_t + \log\left[1 + x_t\left(\exp(r_{t+1}) - 1\right)\right] + \log(1-\gamma), \tag{A.5}$$

albeit a transcendental equation with respect to r. An approximative solution can be obtained by performing a Taylor expansion of Eq (A.5) with respect to r to obtain

$$\log(W_{t+1}) = \log(W_t) + x_t r_{t+1}\left(1 + \frac{r_{t+1}}{2}\right) - \frac{{x_t}^2}{2} r_{t+1}^2 + \log(1-\gamma) + \mathcal{O}\left(r^3\right). \tag{A.6}$$

Substituting Eq. (A.6) into Eq. (A.3), and furthermore approximating $\mathcal{E}(r_{t+1}^2)$ with $\mathrm{Var}(r_{t+1})$ we obtain

$$argmax_{D_t \in [0,\lambda_{\max}]} \left\{ \log W_t + x_t \mathcal{E}(r_{t+1}) + \frac{x_t}{2}(1-x_t)\mathrm{Var}(r_{t+1}) + \log(1-\gamma) \right\}. \tag{A.7}$$

Finally the first-order condition yields

$$x_t = \min \left[\frac{\mathcal{E}(r_{t+1}) + \frac{1}{2}a\mathrm{Var}(r_{t+1})}{a\mathrm{Var}\ (r_{t+1})}, \lambda_{\max} \right]. \tag{A.8}$$

Consequently, the optimal demand for HF j in terms of the number of shares of the risky asset given the price at the current period is

$$D_t = \min \left\{ \frac{\log(V/p_t) + \frac{1}{2}a\mathrm{Var}\left[\log p_{t+1}|\mathcal{F}^j\right]}{a\mathrm{Var}\left[\log p_{t+1}|\mathcal{F}^j\right]}, \lambda_{\max} \right\} \frac{W_t}{p_t}. \tag{A.9}$$

3.7 Appendix B

As already stated in Section 3.4.2, Theorem 3, assuming that the process defined by $Y(t) = 1_{[0,t]}(S_m)$ is ergodic, the auto-correlation function can be expressed as a time-average

$$C(t') \propto \lim_{K\to\infty} \frac{1}{K} \sum_{t=0}^{K} Y_t Y_{t+t'}. \tag{B.1}$$

Given that $Y(t)$ is by definition a binary variable, the only non-zero terms contributing to the sum appearing on the right hand side (RHS) of equation (B.1) correspond to default events (mapped to 1) that occur with a time difference equal to t'. Therefore, the RHS of equation (B.1) is proportional to the conditional probability of observing a default at time t', given that a default has occurred at time $t = 0$. Therefore, we can express $C(t')$ in terms of the aggregate probability $\tilde{P}(\tau = t')$, i.e. the probability of a default event being observed after t' time-steps, given that one has just been observed. Moreover, we must take into account all possible combinations of defaults happening at times $t < t'$. For example, let us assume that we want to calculate $C(t' = 2)$. In this case there are exactly 2 possible set of events that would give a non-zero contribution. Either a default happening exactly 2 time-steps after the last one (at $t = 0$), or two subsequent defaults happening at $t = 1$, and $t = 2$. In this fashion, we can express the correlation function in terms of the probability the waiting-times between defaults as [Procaccia and Schuster, 1983],

$$\begin{aligned}
C(1) &= \tilde{P}(1), && \text{(B.2)}\\
C(2) &= \tilde{P}(2) + \tilde{P}(1)\tilde{P}(1) \\
&= \tilde{P}(2) + \tilde{P}(1)C(1), && \text{(B.3)}\\
&\vdots \\
C(t') &= \tilde{P}(t') + \tilde{P}(t'-1)C(1) + \ldots \tilde{P}(1)C(t'-1). && \text{(B.4)}
\end{aligned}$$

If we further define $C(0) = 1$ and $\tilde{P}(0) = 0$, then equation (B.4) can be written more compactly as

$$C(t') = \sum_{\tau=0}^{t'} C(t'-\tau)\tilde{P}(\tau) + \delta_{t',0}, \tag{B.5}$$

where $\delta_{t',0}$ is the Kronecker delta.

We are interested only in the long time limit of the ACF. Hence, we can treat time as a continuous variable and solve equation (B.5) by applying the Laplace transform $\mathcal{L}\{f(\tau)\}(s) = \int_0^\infty f(\tau)\exp(-s\tau)d\tau$, utilising also the convolution theorem .Taking these steps we obtain

$$C(s) = \frac{1}{1 - \tilde{P}(s)}, \tag{B.6}$$

where $\tilde{P}(s) = \mathcal{L}\left\{\tilde{P}(\tau)\right\}(s) = \int_0^\infty \tilde{P}(\tau)\exp(-s\tau)d\tau$. We will assume that $\tilde{P}(\tau) \propto \tau^{-a}$ for any $\tau \in [1, \infty)$, i.e. the asymptotic power-law behaviour ($\tau \gg 1$) will be assumed to remain accurate for all values of τ. Under this assumption,

$$\tilde{P}(\tau) = \begin{cases} A\tau^{-a}, & \tau \in [1, \infty), \\ 0, & \tau \in [0, 1). \end{cases}, \tag{B.7}$$

where $A = 1/\int_1^\infty \tau^{-a}d\tau = a - 1$. The Laplace transform of equation (B.7) is,

$$\tilde{P}(s) = (a-1)E_a(s), \tag{B.8}$$

where $E_a(s)$ denotes the exponential integral function defined as,

$$E_a(s) = \int_1^\infty \exp\left(-st\right) t^{-a} dt \ /; \ \mathrm{Re}(s) > 0. \tag{B.9}$$

The inversion of the Laplace transform after the substitution of equation (B.8) in equation (B.6) is not possible analytically. However, we can easily derive the correlation function in the Fourier space (known as the power spectral density function) $\mathcal{F}\{C(t')\}(f) = \sqrt{\frac{2}{\pi}} \int_0^\infty C(t')\cos(2\pi f)dt'$ by the use of the identity [Jeffrey and Zwillinger, 2007, p. 1129],

$$\mathcal{F}\{C(t')\} = \frac{1}{\sqrt{2\pi}}\left[C(s \to 2\pi i f) + C(s \to -2\pi i f)\right]. \tag{B.10}$$

relating the Fourier cosine transform $\mathcal{F}\left\{g(t)\right\}(f)$, of a function $g(t)$, to its Laplace transform $g(s)$, to obtain,

$$C(f) = \frac{1}{\sqrt{2\pi}}\left(\frac{1}{1-(a-1)E_a(2if\pi)} + \frac{1}{1-(a-1)E_a(-2if\pi)}\right) \tag{B.11}$$

From equation (B.11) we can readily see that as $f \to 0_+$ (equivalently $t' \to \infty$), $C(f) \to \infty$. To derive the asymptotic behaviour of $C(f)$ we expand about $f \to 0_+$ (up to linear order) using

$$E_a(2if\pi) = ai^{a+1}(2\pi)^{a-1}f^{a-1}\Gamma(-a) - \frac{2i\pi f}{a-2} + \frac{1}{a-1} + O(f^2) \tag{B.12}$$

to obtain

$$
\begin{aligned}
C(f) = &-\frac{i\sqrt{2\pi}(a-2)f}{4\pi^2(a-1)f^2 + \left(2^{a+1}\pi^a(if)^a - a(2i\pi)^a f^a\right)\Gamma(2-a)} \\
&+ \frac{i\sqrt{2\pi}(a-2)f}{4\pi^2(a-1)f^2 + \left(2^{a+1}\pi^a(-if)^a - a(-2i\pi)^a f^a\right)\Gamma(2-a)}.
\end{aligned}
\tag{B.13}
$$

After some algebraic manipulation, for $f \to 0$ equation (B.13) yields

$$
C(f) = Af^{a-3}, \tag{B.14}
$$

where

$$
A = -\frac{2^{a+\frac{1}{2}}(a-2)^2\pi^{a-\frac{3}{2}}\sin\left(\frac{\pi a}{2}\right)\Gamma(1-a)}{(a-1)}. \tag{B.15}
$$

Therefore, for $2 < a < 3$ we see that the Fourier transform of the correlation function behaves as,

$$
C(f) \propto f^{a-3}. \tag{B.16}
$$

If $a = 3$, then the instances of the Gamma function appearing on the RHS of equation (B.13) diverge. Therefore, for $a = 3$ we need to use a different series expansion around $f \to 0_+$. Namely,

$$
E_3(2\pi if) = \frac{1}{2} - 2i\pi f + \pi^2 f^2(2\log(2i\pi f) + 2\gamma - 3) + O\left(f^5\right), \tag{B.17}
$$

where γ stands for the Euler's constant. The substitution of equation (B.17) into equation (B.11) leads to

$$
\begin{aligned}
C(f) = -\operatorname{Re}\Big\{ &[2\log(\pi f) - 2\gamma + 3 - \log(4)] / [\sqrt{2\pi}(2i\pi f\log(\pi f) \\
&+ \pi f(2i\gamma + \pi + i(\log(4) - 3)) - 2) \\
&\times (\pi(3i - 2i\gamma + \pi)f - 2i\pi f\log(2\pi f) - 2)]\Big\},
\end{aligned}
\tag{B.18}
$$

and thus,

$$
\begin{aligned}
C(f) =\Big(&- 8\gamma^3\pi^2 f^2 - 2\pi^2 f(f(-6\log(\pi)(\log\left(16\pi^3\right) \\
&- 2\gamma\log\left(4\pi f^2\right)) + \left(12\gamma^2 + \pi^2\right)\log(\pi f) + 9(3-4\gamma)\log(2\pi f)) \\
&+ 4f\log^3(f) + 6f(2\gamma - 3 + \log(4) + 2\log(\pi))\log^2(f) \\
&+ 6f\left(\gamma\log(16) + (\log(2\pi)-3)\log\left(4\pi^2\right)\right)\log(f) + 4f\log(2\pi)((\log(2)-3)\log(2) \\
&+ \log(\pi)\log(4\pi)) - 4\log(2\pi f)) - 4\gamma^2\pi^2 f^2(\log(64)-9) - 2\gamma(\pi^2 f(f\left(\pi^2 + 27 + 12\log^2(2)\right) \\
&- 4) + 4) + \pi^2 f\left(f\left(27 - \pi^2(\log(4)-3) + \log(8)\log(16)\right) - 12\right) - 8\log(2\pi f) + 12\Big) \\
&\Big/\Big(\sqrt{2\pi}(4\pi^2 f^2\log(\pi f)(\log(4\pi f) + 2\gamma - 3) + \pi^2 f(f(4\gamma^2 + \pi^2 + (\log(4)-3)^2 \\
&+ 4\gamma(\log(4)-3)) - 4) + 4)^2\Big).
\end{aligned}
\tag{B.19}
$$

As $f \to 0$ we have,

$$
C(f) \sim |\log(f)| \tag{B.20}
$$

Finally, if $a > 3$, then equation (B.11) for $f \to 0$ tends to a constant, and thus, Y_t behaves as white noise. Consequently, if the variance of $\tilde{P}(\tau)$ is finite, then Y_t is for large values of t' is uncorrelated.

To summarise, the spectral density function for $f \ll 1$ is,

$$
C(f) \overset{f \ll 1}{\propto} \begin{cases} f^{a-3}, & 2 < a < 3 \\ |\log(f)|, & a = 3 \\ \text{const.}, & a > 3 \end{cases} . \tag{B.21}
$$

The inversion of the Fourier (cosine) transform in equation (B.21) yields,

$$
C(t') \propto t'^{2-a}/;\ 2 < a \leq 3 \wedge t' \gg 1. \tag{B.22}
$$

Bibliography

Andrew B. Abel. Asset prices under habit formation and catching up with the joneses. *American Economic Review*, 80(2):38–42, 1990.

Andrew B. Abel. Risk premia and term premia in general equilibrium. Technical report, 1998.

Viral V. Acharya and S. Viswanathan. Leverage, Moral Hazard, and Liquidity. *Journal of Finance*, 66(1):99–138, 2011.

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, 2010.

Torben G. Andersen and Tim Bollerslev. Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. *The Journal of Finance*, 52(3):975–1005, 1997.

Andrew Ang, Sergiy Gorovyy, and Gregory B. van Inwegen. Hedge fund leverage. *Journal of Financial Economics*, 102(1):102 – 126, 2011.

Richard Arneson. Equality and and equality of opportunity for welfare. *Philosophical Studies*, 56:77–93, 1989.

Kenneth J. Arrow. Rawls's principle of just saving. *The Swedish Journal of Economics*, 75(4):323–335, 1973.

Richard T. Baillie. Long memory processes and fractional integration in econometrics. *Journal of Econometrics*, 73(1):5 – 59, 1996.

Alternative Investment Databases BarclayHedge. The eurekahedge report, 2016. Accessed: 2016-7-07.

Gary S. Becker and Kevin M. Murphy. A theory of rational addiction. *Journal of Political Economy*, 96(4):675–700, 1988.

Gary S. Becker, Kevin M. Murphy, and Ivan Werning. The Equilibrium Distribution of Income and the Market for Status. *Journal of Political Economy*, 113(2):282–310, 2005.

Frank Beichelt. *Stochastic processes in science, engineering and finance.* CRC Press, 2010.

Jeremy Bentham. *The Collected Works of Jeremy Bentham: Official Aptitude Maximized, Expense Minimized.* Clarendon Press, Oxford, 1993.

Jeremy Bentham. *The Works of Jeremy Bentham: Published under the Superintendence of His Executor, John Bowring. Volume 3.* Adamant Media Corporation, 2001b.

Juliana Bidadanure. On dennis mckerlie's 'equality and time'. *Ethics*, 125:1174–1177, 2015.

Juliana Bidadanure. Making sense of age- group justice: A time of relational equality? *Politics, Philosophy & Economics*, 15:234–260, 2016.

Walter Bossert. Redistribution mechanisms based on individual characteristics. *Mathematical Social Sciences*, 29:1 – 17, 1995.

Marcel Boyer. A habit forming optimal growth model. *International Economic Review*, 19(3):585–609, 1978.

Nicole M. Boyson, Christof W. Stahel, and Rene M. Stulz. Hedge fund contagion and liquidity shocks. *Journal of Finance*, 65(5):1789–1816, 2010.

William A. Brock and Cars H. Hommes. A rational route to randomness. *Econometrica*, 65(5):1059–1096, 1997.

William A. Brock and Cars H. Hommes. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22 (8-9):1235–1274, 1998.

William A Brock and Blake D. LeBaron. A Dynamic Structural Model for Stock Return Volatility and Trading Volume. *The Review of Economics and Statistics*, 78(1):94–110, 1996.

Markus K. Brunnermeier and Lasse H. Pedersen. Market Liquidity and Funding Liquidity. *Review of Financial Studies, Society for Financial Studies*, 22(6):2201–2238, 2009.

Markus K. Brunnermeier and Yuliy Sannikov. A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104(2):379–421, 2014.

John Y. Campbell and John H. Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *National Bureau of Economic Research, Inc*, 107, 1995.

John Y. Campbell and Luis M. Viceira. *Strategic asset allocation: portfolio choice for long-term investors.* Oxford University Press, USA, 2002.

Christopher D. Carroll. Solving consumption models with multiplicative habits. *Economics Letters*, 68(1):67–77, 2000.

Raj Chetty, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang. The Fading American Dream: Trends in Absolute Income Mobility Since 1940. NBER Working Papers 22910, National Bureau of Economic Research, Inc, 2016.

Carl Chiarella. The dynamics of speculative behaviour. *Annals of operations research*, 37(1):101–123, 1992.

Carl Chiarella and Corrado Di Guilmi. The financial instability hypothesis: A stochastic microfoundation framework. *Journal of Economic Dynamics and Control*, 35(8):1151–1171, 2011.

Carl Chiarella and Xue-Zhong He. Heterogeneous Beliefs, Risk and Learning in a Simple Asset Pricing Model. *Computational Economics*, 19(1):95–132, 2002.

Carl Chiarella, Roberto Dieci, and Xue-Zhong He. Heterogeneity, market mechanisms, and asset price dynamics. In T. Hens and K. R. Schenk-Hoppe, editors, *Handbook of Financial Markets: Dynamics and Evolution*, pages 277–344. Elsevier, 2009.

Carl Chiarella, Xue-Zhong He, and Remco C.J. Zwinkels. Heterogeneous expectations in asset pricing: Empirical evidence from the S&P500. *Journal of Economic Behavior & Organization*, 105(C):1–16, 2014.

Yuan K. Chou. Three simple models of social capital and economic growth. *Journal of Behavioral and Experimental Economics (formerly The Journal of Socio-Economics)*, 35(5):889–912, 2006.

Andrew E. Clark and Andrew J. Oswald. Satisfaction and comparison income. *Journal of Public Economics*, 61(3):359–381, 1996.

Gerald A. Cohen. On the currency of egalitarian justice. *Ethics*, 99:906–944, 1989.

Harold L Cole, George J Mailath, and Andrew Postlewaite. Social Norms, Savings Behavior, and Growth. *Journal of Political Economy*, 100(6):1092–1125, 1992.

James S. Coleman. Social Capital in the Creation of Human Capital. *American Journal of Sociology*, 94, Supplement: Organizations and Institutions: Sociological and Economic Approaches to the Analysis of Social Structure:S95–S120, 1988.

James S. Coleman. *Equality and Achievement in Education*. Westview Press, Boulder, CO, 1990.

James S. Coleman. *Foundations of Social Theory*. Belknap Press, Cambridge, MA, 1994.

George M. Constantinides. Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy*, 98(3):519–43, 1990.

Giacomo Corneo and Olivier Jeanne. Pecuniary Emulation, Inequality and Growth. *European Economic Review*, 43(9):1665–1678, 1999.

Norman Daniels. *Am I my Parents' Keeper?* Oxford University Press, New York, 1988.

Norman Daniels. The prudential lifespan account: Objections and replies. In L. Cohen, editor, *Justice Across Generations: What Does It Mean?* American Association of Retired People, Washington, D.C, 1993.

Norman Daniels. Justice Between Adjacent Generations: Further Thoughts. *Journal of Political Theory*, 16:475–494, 2008.

Partha Dasgupta. On some Alternative Criteria for Justice between Generations. *Journal of Public Economics*, 3(4):405–423, 1974.

Richard H. Day and Weihong Huang. Bulls, bears and market sheep. *Journal of Economic Behavior & Organization*, 14(3):299–329, 1990.

Angus Deaton. *Understanding Consumption*. Clarendon Press, Oxford, 1992.

Lokenath Debnath and Dambaru Bhatta. *Integral transforms and their applications*. Chapman & Hall/CRC, 2007.

Corrado Di Guilmi, Xue-Zhong He, and Kai Li. Herding, trend chasing and market volatility. *Journal of Economic Dynamics and Control*, 48(C):349–373, 2014.

James Duesenberry. *Income, Saving and the Theory of Consumer Behavior*. Oxford University Press, New York, 1949.

Ronald Dworkin. What is equality? part 2: Equality of resources. *Philosophy and Public Affairs*, 10:283–345, 1981.

Doyne Farmer and John Geanakoplos. Power laws in economics and elsewhere. 2008.

Wayne E. Ferson and George M. Constantinides. Habit persistence and durability in aggregate consumption: Empirical tests. *Journal of Financial Economics*, 29 (2):199–240, 1991.

Irvin Fisher. *The Theory of Interest.* Macmillan, New York, 1930.

Marc Fleurbaey. Three solutions for the compensation problem. *Journal of Economic Theory*, 65:505– 521, 1995.

Marc Fleurbaey. *Fairness, Responsibility and Welfare.* Oxford University Press, 2008.

Marc Fleurbaey and Francois Maniquet. *A Theory of Fairness and Social Welfare.* Cambridge University Press, 2011.

Ana Fostel and John Geanakoplos. Leverage cycles and the anxious economy. *American Economic Review*, 98(4):1211 – 1244, 2008. ISSN 00028282.

Ana Fostel and John Geanakoplos. Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes. *American Economic Journal: Macroeconomics*, 4(1):190–225, 2012.

Ana Fostel and John Geanakoplos. Endogenous collateral constraints and the leverage cycle. *Annual Review of Economics*, 6(1):771–799, 2014.

Ana Fostel and John Geanakoplos. Leverage and Default in Binomial Economies: A Complete Characterization. *Econometrica*, 83(6):2191–2229, 2015.

Ana Fostel and John Geanakoplos. Financial innovation, collateral, and investment. *American Economic Journal: Macroeconomics*, 8(1):242–84, 2016.

Robert Frank. *Choosing the Right Pond: Human Behavior and the quest for status.* Oxford University Press, New York, 1985.

Giorgos Galanis and Roberto Veneziani. Equality of *When*? *Œconomia – History / Methodology / Philosophy*, 7(1), 2017.

Jordi Gali. Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices. *Journal of Money, Credit and Banking*, 26(1):1–8, 1994.

John Geanakoplos. Promises Promises. Technical report, 1996.

John Geanakoplos. Liquidity, default, and crashes endogenous contracts in general. In *Advances in economics and econometrics: theory and applications: eighth World Congress*, volume 170, 2003.

John Geanakoplos. The leverage cycle. In *NBER Macroeconomics Annual 2009, Volume 24*, pages 1–65. National Bureau of Economic Research, Inc, 2010.

John Geanakoplos. Leverage, Default, and Forgiveness: Lessons from the American and European Crises. *Journal of Macroeconomics*, 39(PB):313–333, 2014.

John Geanakoplos and William Zame. Collateral and the enforcement of intertemporal contracts. Cowles foundation discussion papers, Cowles Foundation for Research in Economics, Yale University, 1997.

John Geanakoplos and William Zame. Collateral equilibrium, I: a basic framework. *Economic Theory*, 56(3):443–492, 2014.

Edward L. Glaeser, David Laibson, and Bruce Sacerdote. An Economic Approach to Social Capital. *Economic Journal*, 112(483):437–458, 2002.

Greg N Gregoriou, Georges Hübner, Nicolas Papageorgiou, and Fabrice D Rouah. *Hedge funds: Insights in performance measurement, risk analysis, and portfolio allocation*, volume 313. John Wiley & Sons, 2007.

Denis Gromb and Dimitri Vayanos. Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2-3):361–407, 2002.

Shermineh Haghani. Modeling hedge fund lifetimes: A dependent competing risks framework with latent exit types. *Journal of Empirical Finance*, 2014.

Xue-Zhong He and Youwei Li. Power-law behaviour, heterogeneity, and trend chasing. *Journal of Economic Dynamics and Control*, 31(10):3396–3426, 2007.

Brad Hershbein. A college degree is worth less if you are raised poor. Social mobility memos, Brookings, 2016.

Cars H. Hommes. Heterogeneous agent models in economics and finance. In L. Tesfatsion and K. L. Judd, editors, *Handbook of Computational Economics, Volume 2*, pages 1109 – 1186. Elsevier, 2006.

Giulia Iori. A microsimulation of traders activity in the stock market: the role of heterogeneity, agents' interactions and trade frictions. *Journal of Economic Behavior & Organization*, 49(2):269–285, 2002.

Alan Jeffrey and Daniel Zwillinger. *Table of Integrals, Series, and Products*. Table of Integrals, Series, and Products Series. Elsevier Science, 2007. ISBN 9780080471112.

Colin Jennings and Santiago Sanchez-Pages. Social capital, conflict and welfare. *Journal of Development Economics*, 124(C):157–167, 2017.

Harl E. Ryder Jr and Geoffrey M. Heal. Optimum growth with intertemporally dependent preferences. *Review of Economic Studies*, 40(1):1–33, 1973.

Laura E. Kodres and Matthew Pritsker. A Rational Expectations Model of Financial Contagion. *Journal of Finance*, 57(2):769–799, 2002.

Albert S. Kyle and Wei Xiong. Contagion as a Wealth Effect. *Journal of Finance*, 56(4):1401–1440, 2001.

Blake D. LeBaron. Agent-based computational finance. In Leigh Tesfatsion and Kenneth L. Judd, editors, *Handbook of Computational Economics*, volume 2 of *Handbook of Computational Economics*, chapter 24, pages 1187–1233. Elsevier, 2006.

Abba P. Lerner. *The Economics of Control: Principles of Welfare Economics*. The Macmillan Co, 1944.

Moshe Levy. Stock market crashes as social phase transitions. *Journal of Economic Dynamics and Control*, 32(1):137–155, 2008.

Glenn C. Loury. A Dynamic Theory of Racial Income Differences. Discussion Papers 225, Northwestern University, Center for Mathematical Studies in Economics and Management Science, June 1976.

Thomas Lux. Herd behaviour, bubbles and crashes. *Economic Journal*, 105(431): 881–96, 1995.

Thomas Lux. The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions. *Journal of Economic Behavior & Organization*, 33(2):143–165, 1998.

Thomas Lux and Michele Marchesi. Scaling and criticality in a stochastic multi-agent model of a financial market. *Nature*, 397(6719):498–500, 1999.

Benoit B. Mandelbrot. *The fractal geometry of Nature*. W. H. Freeman and Company, New York, 1983.

Dennis McKerlie. Equality and time. *Ethics*, 99:475–491, 1989.

Dennis McKerlie. Dimensions of equality. *Utilitas*, 3:263–288, 2001a.

Dennis McKerlie. Justice between the young and the old. *Philosophy and Public Affairs*, 30:152–177, 2001b.

Dennis McKerlie. *Justice Between the Young and the Old*. Oxford University Press, New York, 2012.

George Messinis. Habit formation and the theory of addiction. *Journal of Economic Surveys*, 13(4):417–42, 1999.

Omer Moav and Zvika Neeman. Status and Poverty. *Journal of the European Economic Association*, 8(2-3):413–420, 2010.

Roger B Myerson. Utilitarianism, egalitarianism, and the timing effect in social choice problems. *Econometrica*, 49(4):883–97, 1981.

Randolph Nelson. *Probability, Stochastic Processes, and Queueing Theory: The Mathematics of Computer Performance Modeling*. Springer, 1995.

Oxfam. Just 8 men own same wealth as half the world. `https://www.oxfam.org/en/pressroom/pressreleases/2017-01-16/just-8-men-own-same-w` 2016. Accessed: 13/3/2017.

Bhikhu Parekh. Bentham's theory of equality. *Political Studies*, 18(4):478–495, 1970.

Thomas Piketty. Social Mobility and Redistributive Politics. *The Quarterly Journal of Economics*, 110(3):551–84, 1995.

Thomas Piketty. Self-fulfilling beliefs about social status. *Journal of Public Economics*, 70(1):115–132, 1998.

Thomas Piketty. *Capital in the 21st Century*. Harvard University Press, Cambridge, 2014.

Thomas Piketty and Gabriel Zucman. Capital is Back: Wealth-Income Ratios in Rich Countries 1700?2010. *The Quarterly Journal of Economics*, 129(3):1255–1310, 2014.

Sebastian Poledna, Stefan Thurner, Doyne Farmer, and John Geanakoplos. Leverage-induced systemic risk under Basle II and other credit risk policies. *Journal of Banking & Finance*, 42(C):199–212, 2014.

Itamar Procaccia and Heinz Schuster. Functional renormalization-group theory of universal 1/f noise in dynamical systems. *Physical Review A*, 28:1210, 1983.

John Rae. *The Sociological Theory of Capital*. MacMillan, New York, 1834.

John Rawls. *A Theory of Justice*. Belknap Press of Harvard University Press, 1971.

Debraj Ray and Arthur Robson. Status, Intertemporal Choice, and Risk-Taking. *Econometrica*, 80(4):1505–1531, 2012.

Arthur J Robson. Status, the Distribution of Wealth, Private and Social Attitudes to Risk. *Econometrica*, 60(4):837–857, 1992.

John E. Roemer. *Theories of Distributive Justice*. Harvard University Press, 1996.

John E. Roemer. *Equality of Opportunity*. Harvard University Press, 1998.

John E. Roemer and Roberto Veneziani. What We Owe Our Children, They Their Children,.... *Journal of Public Economic Theory*, 6(5):637–654, 2004.

John E. Roemer and Burak Ünveren. Dynamic equality of opportunity. *Economica*, 84(334):322–343, 2017.

Soon Ryoo. Long waves and short cycles in a model of endogenous financial fragility. *Journal of Economic Behavior & Organization*, 74(3):163–186, 2010.

Emmanuel Saez and Gabriel Zucman. Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data. *The Quarterly Journal of Economics*, 131(2):519–578, 2016.

Paul A. Samuelson. A. p. lerner at sixty. *The Review of Economic Studies*, 31(3): pp. 169–178, 1964.

A. Sen. The welfare basis of real income comparisons: A reply. *Journal of Economic Literature*, 18(4):1547–52, 1980.

Amartya Sen. *Commodities and Capabilities.* Oxford University Press, 1985.

Amartya Sen and Bernard Williams. *Utilitarianism and Beyond.* Cambridge University Press, 1982.

Henry Sidgwick. *The Methods of Ethics.* Macmillan, London, 1907.

Alp Simsek. Belief Disagreements and Collateral Constraints. *Econometrica*, 81(1): 1–53, 2013a.

Alp Simsek. Speculation and risk sharing with new financial assets. *Quarterly Journal of Economics*, 128(3):1365–1396, 2013b.

J.J.C. Smart and Bernard Williams. *Utilitarianism: For and Against.* Cambridge University Press, Cambridge, 1973.

R. M. Solow. Intergenerational Equity and Exhaustible Resources. *Review of Economic Studies*, 41(5):29–45, 1974.

George J. Stigler and Gary S. Becker. De gustibus non est disputandum. *American Economic Review*, 67(2):76–90, 1977.

Larry Temkin. Intergenerational inequality. In P. Laslett and J. S. Fishkin, editors, *Justice between Age Groups and Generations.* Yale University Press, New Haven, 1992.

Larry Temkin. *Inequality.* Oxford University Press, Oxford, 1993.

Stefan Thurner, Doyne Farmer, and John Geanakoplos. Leverage causes fat tails and clustered volatility. *Quantitative Finance*, 12:695,707, 2012.

Dirk Van de gaer. *Equality of opportunity and investment in human capital.* PhD Thesis, KULeuven, 1993.

Dimitri Vayanos and Jiang Wang. Liquidity and asset returns under asymmetric information and imperfect competition. *Review of Financial Studies*, 25(5):1339–1365, 2012.

Thorstein Veblen. *The Theory of the Leisure Class. An Economic Study of Institutions.* George Allen Unwin, London., 1922. (First published, 1899).

Wei Xiong. Convergence trading with wealth effects: an amplification mechanism in financial markets. *Journal of Financial Economics*, 62(2):247–292, 2001.